Hadron Linac

Contents

- **INTRODUCTION AND GENERAL PRINCIPLES**
- **PARTICLE ACCELERATION IN RF FIELD**
- **BEAM DYNAMICS**

  1, Longitudinal dynamics

  2, Transverse dynamics

  3, Space charge effect

  4, Dynamics codes

- **INTRODUCTION AND GENERAL PRINCIPLES**

1) Definition: A linear accelerator (linac) produces energetic particles by acceleration in a ***straight line***. But here only accelerating structures that support ***time-varying*** electric fields.

2)Merits of Linear Structure: Capability for producing high energy, high-intensity charged particle beams of excellent quality.

breakdown, radiation, focusing, resonance, inj&extr. ,CW

3) Two categories: traveling wave and standing wave, induction type is excluded in this text.

Typical traveling-wave**: Disc-loaded wave guide**

Typical standing-wave: **Drift tube resonator**

The RF frequency ranges from MHz to GHz (10^6-10^9Hz)

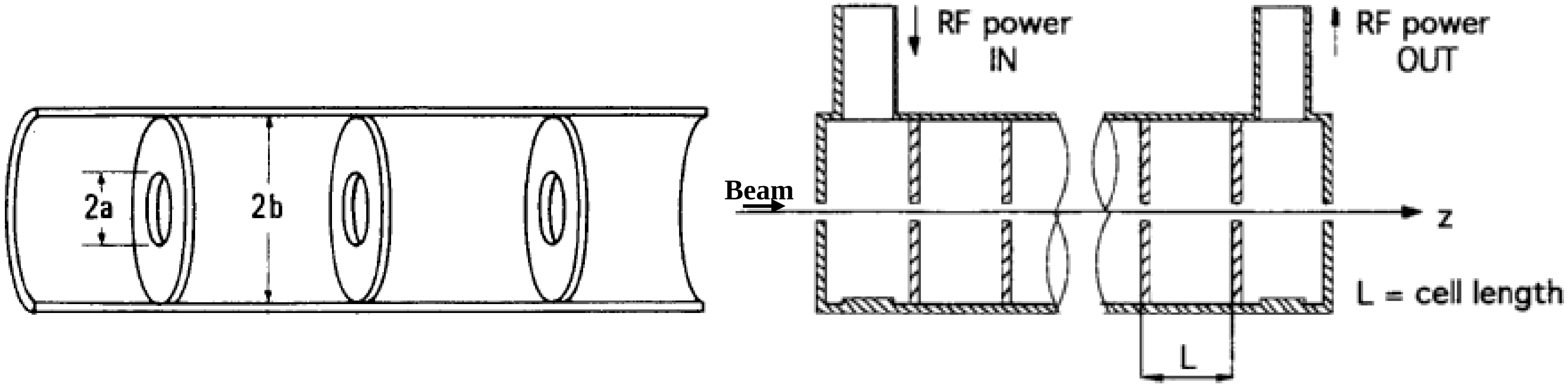


Fig. 1 Disc-loaded traveling-wave linac

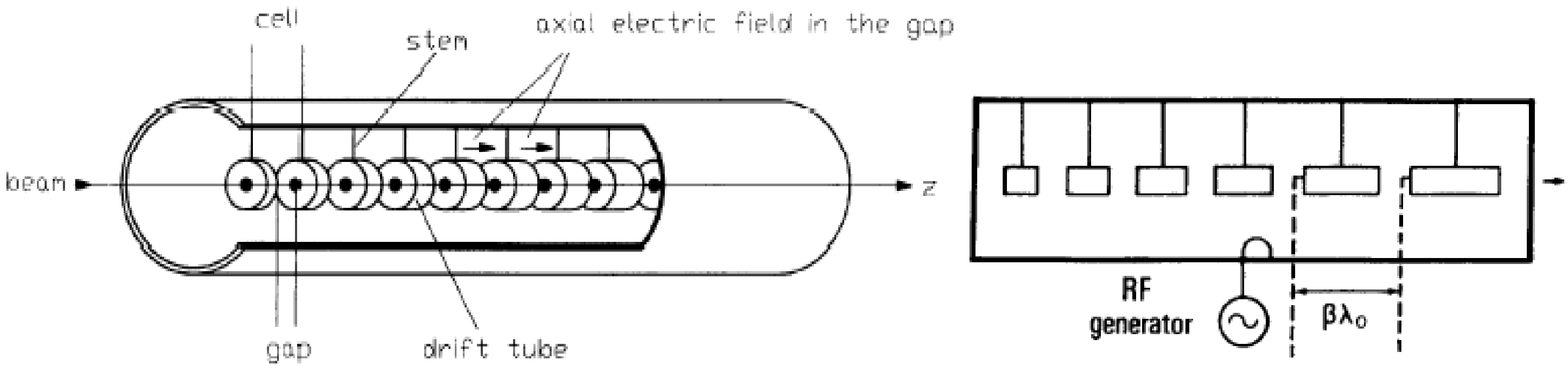


Fig. 2 Alvarez drift tube linac-standing wave structure

- **PARTICLE ACCELERATION IN RF FIELD**

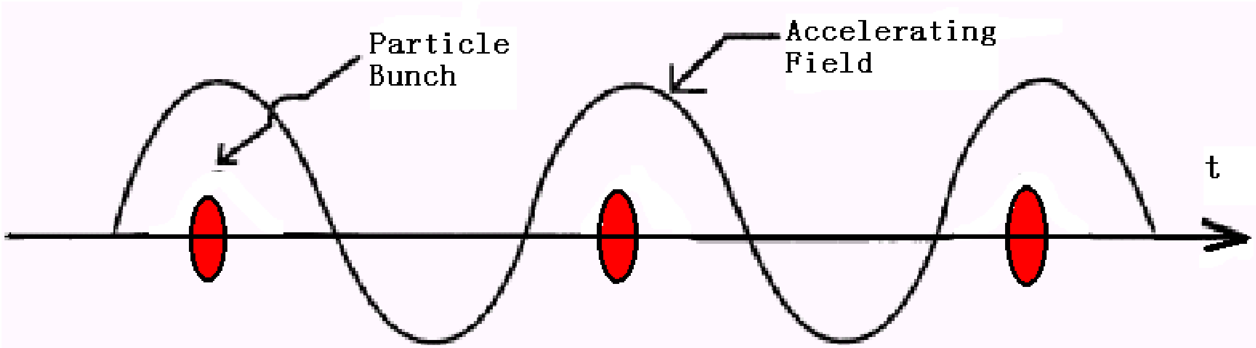


Fig.3 Beam bunch in accelerating field

Traveling-wave accelerating field propagating in +z direction:

$$E_z(z,t) = E(z)\cos\left[\omega t - \int_0^z k(z)dz + \phi\right] \qquad (1)$$

Phase velocity of the wave: $v_p(z) = \frac{\omega}{k(z)}$, $t = .\int_0^z v(z)dz$ To keep acceleration the velocity of a charge q should be always equal to the phase velocity. So the energy gain of the charge in length *L* is:

$$\delta W = q\int_0^L dzE(z)\cos(\phi) = qE_0L\cos(\phi) \qquad (2)$$

The particle is called synchronous particle, and ϕ is called **synchro-nous phase**, noted as ϕ_s. E_0 is the average accelerating field on axis.

In the case of standing-wave, electric filed on axis is :

$$E_z(z,t) = E(z)\cos(\omega t + \phi) \qquad (3)$$

Wave does not propagate while particles moving forward. To avoid the deceleration, tubes are added to screen the particles from the decelerating field, as shown in Fig. 4.

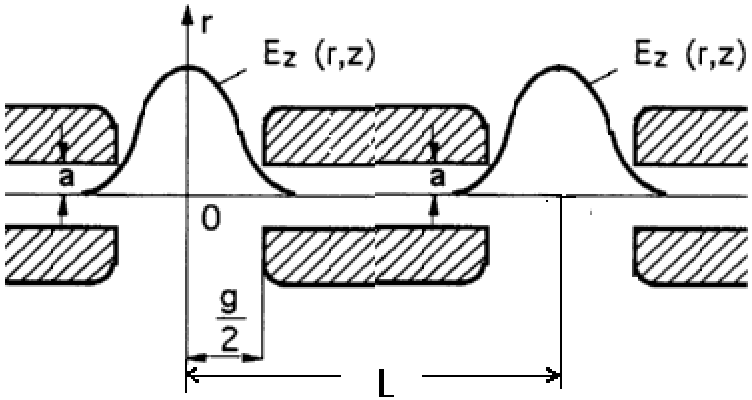


Fig. 4 Accelerating field in the gap between the tubes and the decelerating field is screened within the tubes.

The distance between the two gaps $L=vT=\beta\lambda$, for synchronizing the particle motion with the wave oscillation. In contrast to TW continuous acceleration, the SW acceleration occurred only in the gaps.

A particle of charge q gains energy in one cell of L in length:

$$\delta W = q \int_{-L/2}^{L/2} E(0,z)\cos\left[\omega t(z) + \phi\right]dz \qquad (4)$$

or

$$\delta W = q \int_{-L/2}^{L/2} E(0,z)\left[\cos\omega t\cos\phi - \sin\omega t\sin\phi\right]dz \qquad (5)$$

$$\delta W = qE_0TL\cos\phi \qquad (6)$$

Here E_0 is average axial electric field in length L, and T is defined as

$$T \equiv \frac{\int_{-L/2}^{L/2} E(0,z)\cos\omega t(z)dz}{\int_{-L/2}^{L/2} E(0,z)dz} - \tan\phi\frac{\int_{-L/2}^{L/2} E(0,z)\sin\omega t(z)dz}{\int_{-L/2}^{L/2} E(0,z)dz} \qquad (7)$$

T is called the ***Transit-time Factor*** and E_0T the ***Accelerating Gradient.***

When the change of the particle velocity in the gap is small compared to the initial velocity v, $t \approx z/v$ and then $\omega t \approx 2\pi z/\beta\lambda$.

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/\beta\lambda)\,dz}{\int_{-L/2}^{L/2} E(0,z)\,dz} \qquad (8)$$

In the simple case that the field is in a square profile, as shown in the plot,

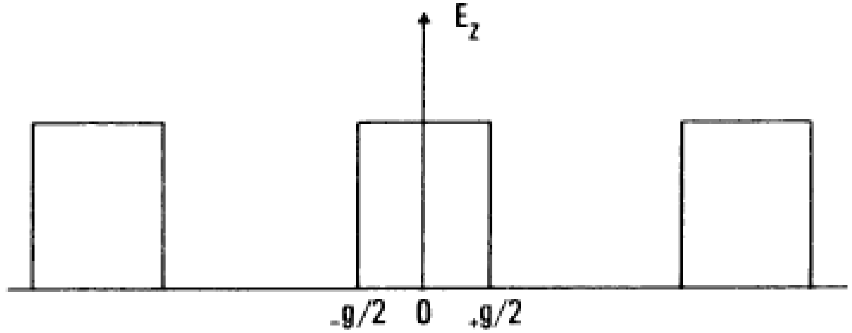
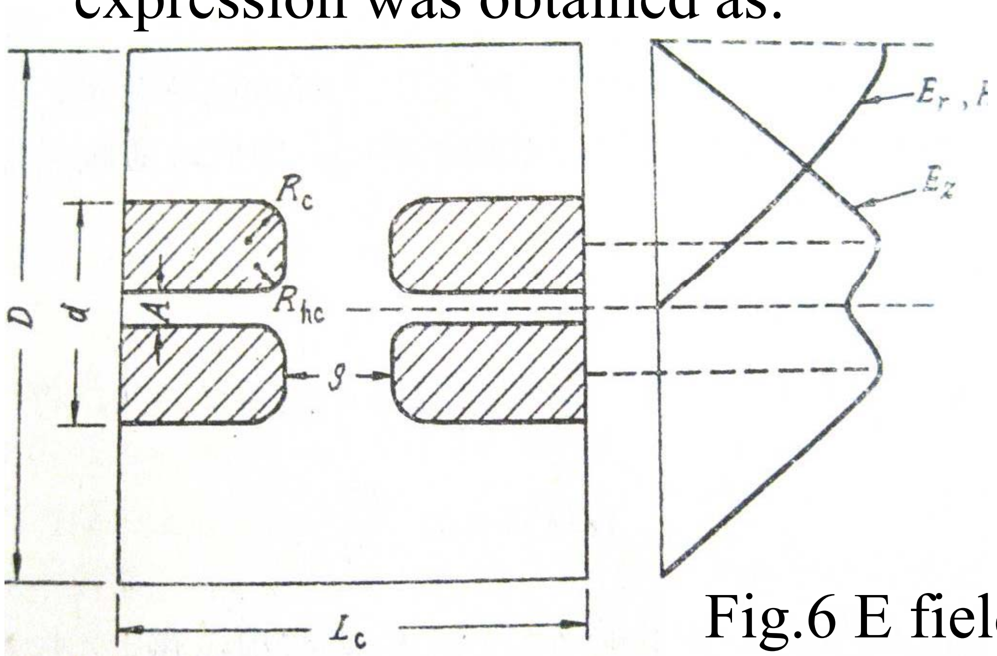


Fig.5

T can be analytically expressed as:

$$T = \frac{\sin(\pi g/\beta\lambda)}{(\pi g/\beta\lambda)} \qquad (9)$$

Considering the field distribution in the transversal and longitudinal directions in a gap with a bore radius of A, an approximated expression was obtained as:



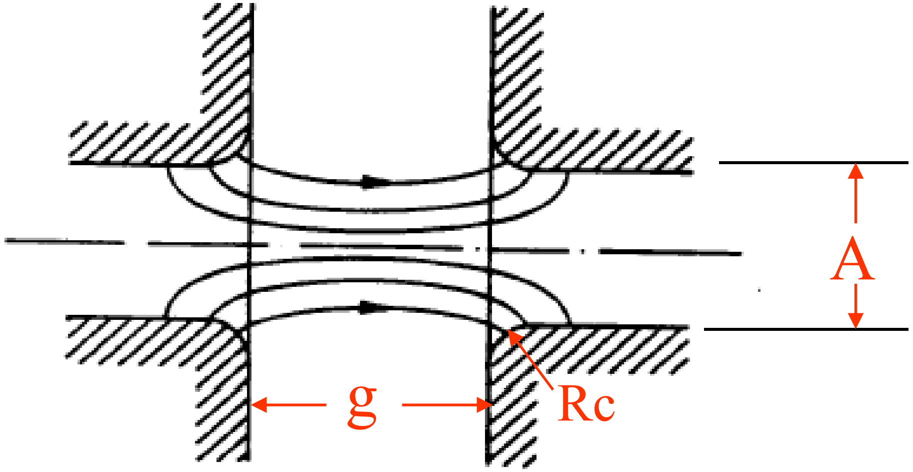


Fig.6 E field distribution near tube

$$T = \frac{\sin(\pi g'/L_c)}{(\pi g'/L_c)} \times \frac{1}{I_0(\frac{2\pi A}{L_c})} \qquad (10)$$

Here g'=g+0.85Rc, with Rc as the inner corner radius of the tube. I_0 is zero-order Bessel function of imaginary argument ($I_0(x) \approx 1 + \frac{x^2}{4}$).

- **BEAM DYNAMICS**

1, Longitudinal dynamics

We need to consider the other particles in a bunch, in reference to the synchronous particle.

$$\delta W - \delta W_s = qE_m L\cos\phi - qE_m L\cos\phi_s$$

($E_m=E_0$ for TW, $E_m=E_0T$ for SW).

$$\frac{d(W-W_s)}{dz} = qE_m(\cos\phi - \cos\phi_s) \qquad (11)$$

$$\frac{d}{dz}(\phi-\phi_s) = \frac{\omega}{c}\left(\frac{1}{\beta}-\frac{1}{\beta_s}\right) \qquad \left(\frac{1}{\beta}-\frac{1}{\beta_s} = -\frac{\gamma-\gamma_s}{(\gamma_s^2-1)^{3/2}} = -\frac{\delta\gamma}{\beta_s^3\gamma_s^3}\right)$$

$$= -\frac{2\pi}{\lambda}\frac{W-W_s}{mc^2}\frac{1}{\beta_s^3\gamma_s^3} \qquad (12)$$

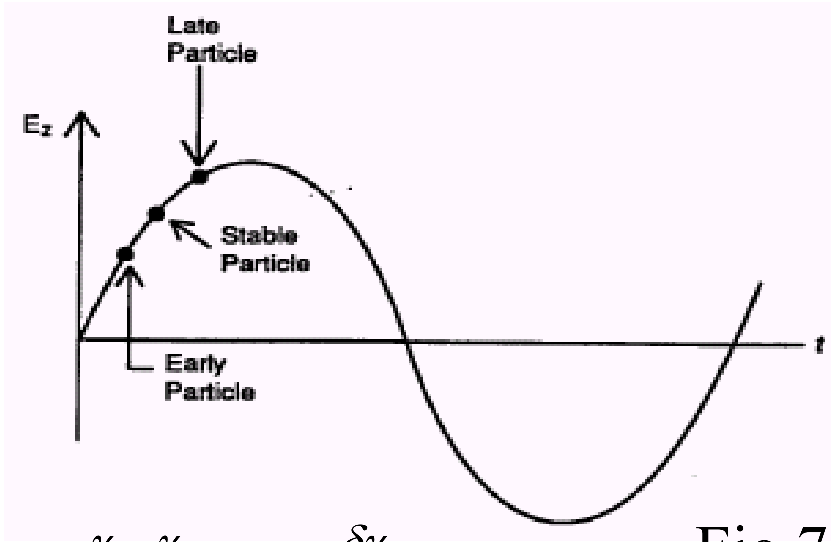


Fig.7

Finally, we have: $$\frac{d}{dz}\left[\beta_s^3\gamma_s^3\frac{d}{dz}\Delta\phi\right] = -\frac{2\pi}{\lambda}\frac{qE_m}{mc^2}(\cos\phi-\cos\phi_s) \qquad (13)$$

After the first integration:

$$\frac{1}{2}\beta_s^3\gamma_s^3\left[\frac{d\Delta\phi}{dz}\right]^2 = -\frac{2\pi}{\lambda}\frac{qE_m}{mc^2}(\sin\phi - \phi\cos\phi_s + C) \quad (14)$$

Substituting Eq.(12) into Eq.(14) yields:

$$\frac{2\pi(\Delta W)^2}{2\beta_s^3\gamma_s^3\lambda mc^2} + qE_m(\sin\phi - \phi\cos\phi_s + C) = 0 \quad (15)$$

C is a constant which defined by the initial condition ($\Delta W_i, \phi_i$). One C value corresponds to a trajectory of a particle in ($\Delta W, \phi$) phase space. In Fig.8, at the top, the accelerating field as function of phase, the synchronous energy gain Ws shown as a broken line. Next, phase-plane trajectories including the separatrix, and at bottom the effective potential well.

(RF Bucket)

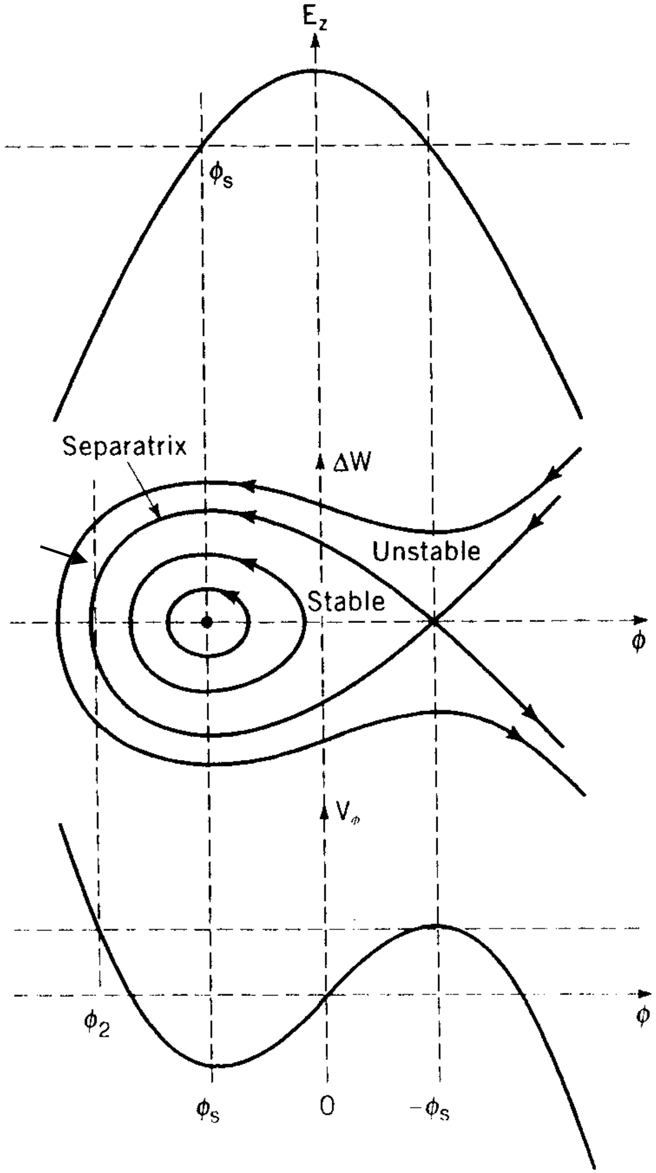


Fig. 8

Longitudinal focusing requires ϕ_s<0.

The separatrix intersects the positive side of the ϕ-axis at the point ϕ_{max}=- ϕ_s, where ϕ_s <0. At this point, we have from Eq.(15):

$$C=\sin\phi_s-\phi_s\cos\phi_s \qquad (16)$$

This C defines the separatrix curve in the Fig. 8. The value ϕ_{min} where the separatrix intersects the negative side of the ϕ-axis can be found by numerical method: $\phi_{min} \approx -2|\phi_s|$, when $|\phi_s| < \pi/3$. So the separatric phase width $\Delta\phi$ is:

$$-2|\phi_s| <= \Delta\phi <= |\phi_s| \qquad (17)$$

Separatrix dependence on synchronous phase:

$\phi_s=0$: $\Delta\phi=0$, δW-max, but rf bucket=0.

$\phi_s=-90^O$: $\Delta\phi$-max:(-3π/2,π/2), but no acceleration.

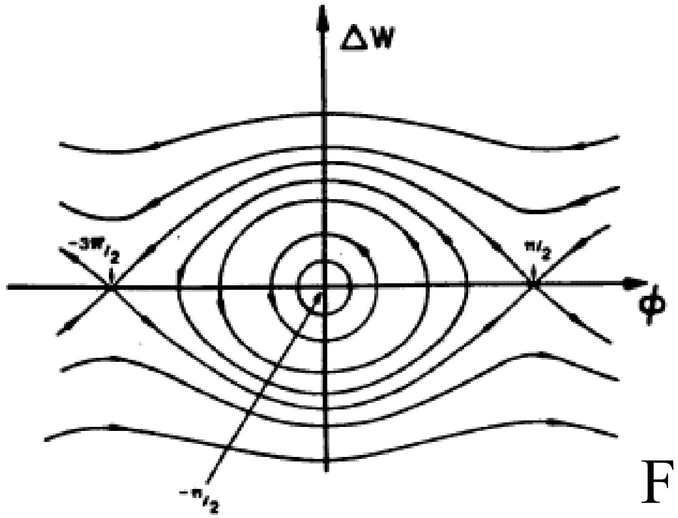


Fig.9

When ϕ=ϕ_s the ΔW reaches its maximum on separatrix:

$$\Delta W_{\max} = 2\left[\beta_s^3\gamma_s^3 \frac{\lambda}{2\pi} mc^2 qE_m(\phi_s\cos\phi_s - \sin\phi_s)\right]^{1/2} \qquad (18)$$

which is the ΔW limit for a particle to be captured in an rf bucket.

ϕ_s=*0:* ΔW_{max} =0, acceptance=0.

ϕ_s=*-90^O: ΔW_{max}-max:* $\Delta W_{\max} = 2\left[\beta_s^3\gamma_s^3 \frac{\lambda}{2\pi} mc^2 qE_m\right]^{1/2}$

ϕ_s selection :

ϕ_s=*-90^O* gives largest acceptance in longitudinal phase space. But no acceleration. (It is used at the injection section of a linac for a DC beam).

Usually, ϕ_s~*-30^O for acceleration regime.*

Let's linearize the motion equation Eq.(13) for the case $\Delta\phi \ll 1$. Making use of the relation $cos(\phi_s+\Delta\phi)-cos\phi_s \approx -\Delta\phi sin\phi_s$, Eq.(13) becomes:

$$\beta_s^3\gamma_s^3\left[\frac{d^2}{dz^2}\Delta\phi\right] = -\frac{2\pi}{\lambda}\left(\frac{qE_m}{mc^2}\sin\phi_s\right)\Delta\phi, \qquad (19)$$

or in harmonic-oscillator form:

$$\frac{d^2(\Delta\phi)}{dz^2} + k_l^2\Delta\phi = 0, \qquad (20)$$

with $k_l = \left[-\frac{2\pi qE_m \sin\phi_s}{\lambda mc^2\beta_s^3\gamma_s^3}\right]^{1/2}$., defined as longitudinal wave-number.

$\omega_l = k_l v_s$ is known as *synchronous frequency.*

From Eq.(20): $\Delta\phi = (\Delta\phi)_m \cos(k_l z + \alpha)$ $\quad (\Delta\phi)_m$, α: on initial conditions

From Eq.(12): $\Delta W = (\Delta W)_m \sin(k_l z + \alpha)$ $\quad (\Delta W)_m = (\Delta\phi)_m \frac{k_l\lambda\beta_s^3\gamma_s^3 mc^2}{2\pi}$

Ellipse equation in *ΔW-Δφ* phase space : $\frac{(\Delta\phi)^2}{(\Delta\phi)_m^2} + \frac{(\Delta W)^2}{(\Delta W)_m^2} = 1$

The area of the ellipse=$\pi(\Delta\phi)_m(\Delta W)_m$=constant , according to Liouville's theorem.

$$(\Delta\phi)_m = \frac{\text{constant}}{\left[qE_m mc^2\beta_s^3\gamma_s^3\lambda\sin(-\phi_s)/2\pi\right]^{1/4}} \quad \text{, or} \quad (\Delta\phi)_m = \frac{\text{constant}}{\left[\beta_s\gamma_s\right]^{3/4}} \qquad (21)$$

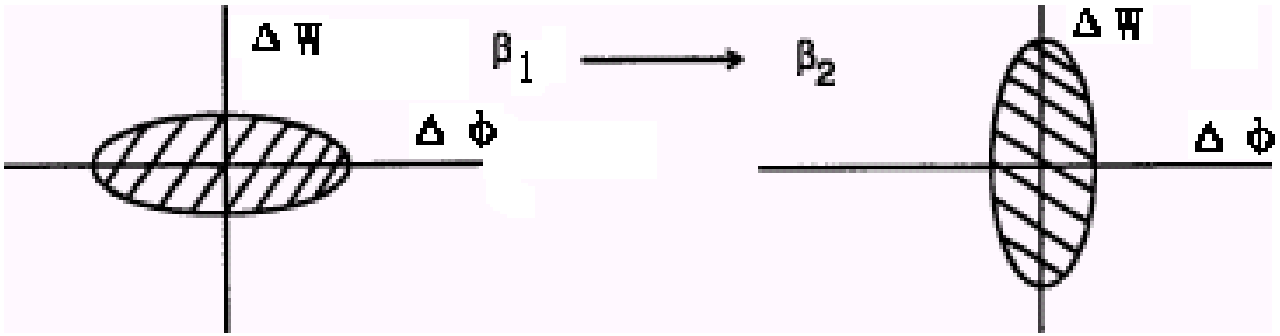


Fig. 10

This is called ***phase damping*** caused by acceleration.

2, Transverse dynamics

(1)Transverse RF defocusing

There is an electric defocusing across the RF accelerating gap for the reason that E(t) increases with t when $\phi_s < 0$.

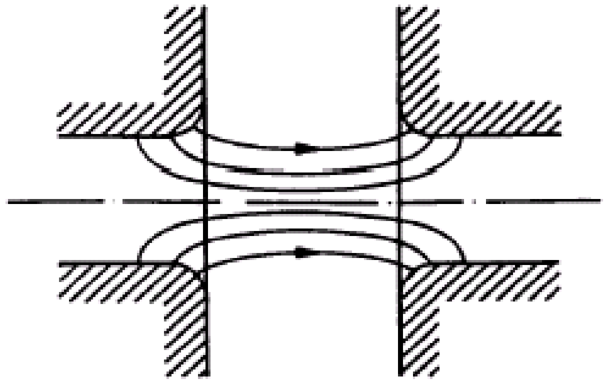


Fig. 11

(2) Field analysis near the axis

Maxwell's Equations

for TM mode:

$$\frac{1}{r}\frac{\partial(rE_r)}{\partial r}+\frac{\partial E_z}{\partial z}=0 \quad \text{from } (\nabla\cdot\bar{E}=0),$$

$$\frac{\partial E_r}{\partial z}-\frac{\partial E_z}{\partial r}=-\frac{\partial B_\theta}{\partial t} \quad \text{from } (\nabla\times\bar{E})_\theta=-\frac{\partial B_\theta}{\partial t},$$

$$-\frac{\partial B_\theta}{\partial z}=\frac{1}{c^2}\frac{\partial E_r}{\partial t} \quad \text{from } (\nabla\times\bar{B})_r=\frac{1}{c^2}\frac{\partial E_r}{\partial t},$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=\frac{1}{c^2}\frac{\partial E_z}{\partial T} \quad \text{from } (\nabla\times\bar{B})_z=\frac{1}{c^2}\frac{\partial E_z}{\partial t}. \qquad (22)$$

Ez is independent of r near axis

From the 1st one: $rE_r = -\frac{\partial E_z}{\partial z}\int_0^r r\,dr$, or $E_r = -\frac{\partial E_z}{\partial z}\frac{r}{2}$. $\frac{\partial E_r}{\partial r} = -\frac{1}{2}\frac{\partial E_z}{\partial z}$.

From the 3rd one: $\frac{\partial B_\theta}{\partial z} = -\frac{1}{c^2}\frac{\partial E_r}{\partial t} = \frac{r}{2c^2}\frac{\partial}{\partial z}\frac{\partial E_z}{\partial t}$. or $B_\theta = \frac{r}{2c^2}\frac{\partial E_z}{\partial t}$.

Assume the standing-wave field for E_z near the axis looks like

$$\mathbf{E_z(z,t) = E_a(z)\cos(\omega t + \phi)} \quad (23)$$

Then the field in r direction can be deduced, as Fig.12.

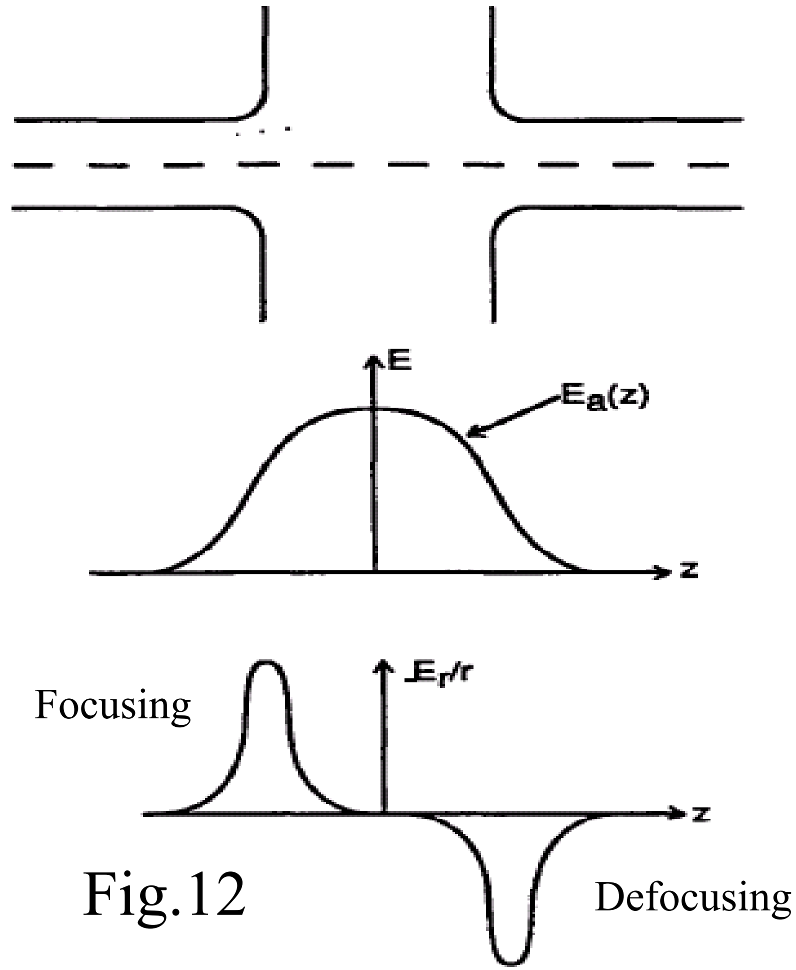


Fig.12

The momentum impulse near the axis is:

$$\Delta p_r = q\int_{-L/2}^{L/2}\left(E_r - \beta c B_\theta\right)\frac{dz}{\beta c} = -\frac{q}{2}\int_{-L/2}^{L/2} r\left[\frac{\partial E_z}{\partial z} + \frac{\beta}{c}\frac{\partial E_z}{\partial t}\right]\frac{dz}{\beta c}.$$

or

$$\Delta p_r = -\frac{qr}{2\beta c}\int_{-L/2}^{L/2}\left[\frac{dE_z}{dz} - \left(\frac{1}{\beta c} - \frac{\beta}{c}\right)\frac{\partial E_z}{\partial t}\right]dz \qquad (24)$$

by making use of the relation: $\frac{dE_z}{dz} = \frac{\partial E_z}{\partial z} + \frac{1}{\beta c}\frac{\partial E_z}{\partial t}$

In Eq.(24) $\frac{dE_z}{dz}$ term vanishes when E_z extends into the zero-field regime in the tube, and thus we have:

β→1, no RF defocusing

$$\Delta p_r = -\frac{qr\omega}{2\beta^2 c^2}\int_{-L/2}^{L/2}(1-\beta^2)E_a(z)\sin(\omega t + \phi)dz \qquad (25)$$

or

$$\Delta p_r = -\frac{qr\omega}{2\beta^2\gamma^2 c^2}\int_{-L/2}^{L/2}E_a(z)\sin(\omega t + \phi)dz$$

Making use of $\sin(\omega t + \phi) = \sin\omega t\cos\phi + \cos\omega t\sin\phi$, and considering $E_a(z)$ is an even function about the gap center where we set t=0, we get

$$\Delta p_r = -\frac{qr\omega}{2\beta^2\gamma^2 c^2}\sin\phi\int_{-L/2}^{L/2} E_a(z)\cos kz\,dz$$

Finally, introducing the definition of E_0 and T, and substituting $p_r = mc\beta\gamma r'$, we obtain

$$\Delta(\gamma\beta r') = -\frac{\pi q E_0 TL \sin\phi}{mc^2\lambda\beta^2\gamma^2} r \qquad (26)$$

This can be written as:

$$\frac{d\beta\gamma x'}{dz} = \frac{\Delta\beta\gamma x'}{L} = -\frac{\pi q E_0 T \sin\phi}{mc^2\lambda\beta^2\gamma^2} x$$

or

$$\frac{1}{\beta\gamma}\frac{d}{dz}\beta\gamma x' - \frac{k_{10}^2}{2} x = 1 \qquad (27)$$

with $\mathbf{k}_{10}^2 = -\frac{2\pi q E_0 TL \sin\phi}{mc^2\lambda\beta^3\gamma^3}$ defined as longitudinal wave-number in Eq.(20)

(3)Quadrupole focusing in a linac

Ideal quadrupole field has a constant field gradient:

$$\mathbf{G} = \frac{\partial \mathbf{B}_x}{\partial \mathbf{y}} = \frac{\partial \mathbf{B}_y}{\partial \mathbf{x}}$$

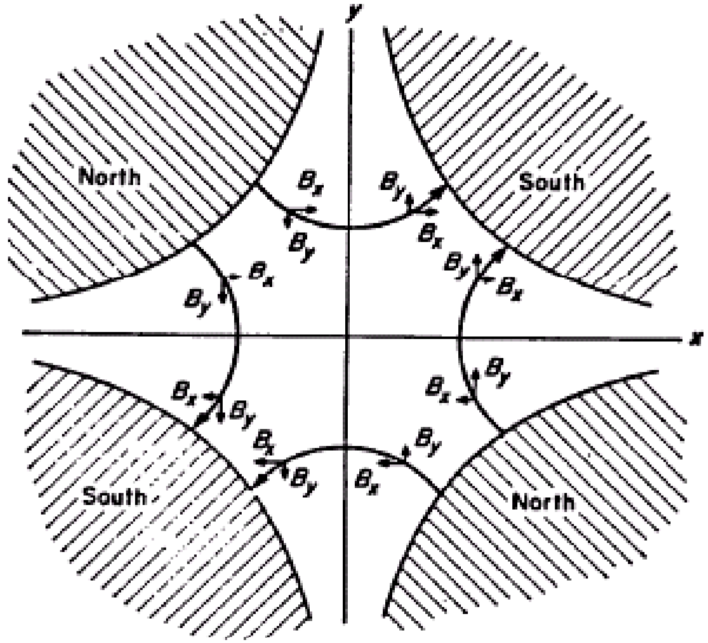


Fig.13 Quadrupole magnet

Lorentz force on a particle moving in z direction at (x,y):

$F_x = -qvGx, \quad F_y = qvGy.$

And the motion equations are: $$\frac{d^2x}{dz^2} + \kappa^2(z)x = 0 \qquad \frac{d^2y}{dz^2} - \kappa^2(z)y = 0$$

where

$$\kappa^2(z) = \frac{|qG(z)|}{mc\beta\gamma} \qquad (28)$$

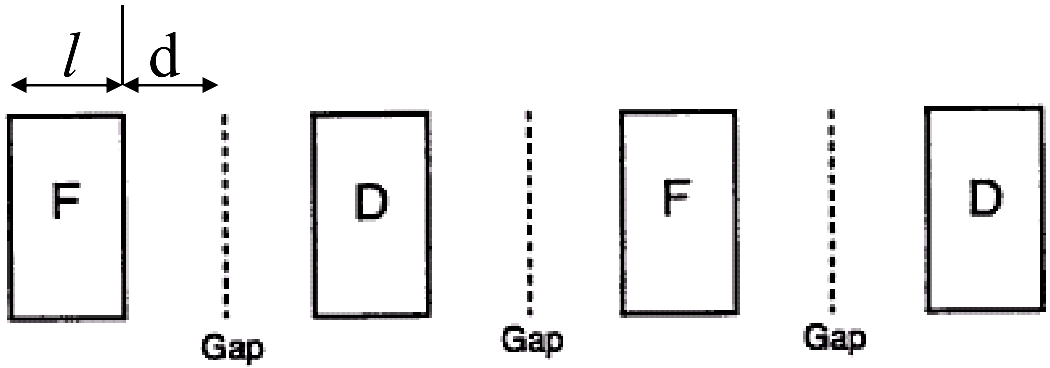


Fig.14 FD lattice with accelerating gaps

Including both quadrupole term and RF defocusing term,we have:

$$\frac{d^2x}{dz^2} + \kappa^2(z)x - \frac{k_{l0}^2}{2}x = 0 \qquad \frac{d^2y}{dz^2} - \kappa^2(z)y - \frac{k_{l0}^2}{2}y = 0 \qquad (29)$$

The equation can be expressed in a normalized form as:

$$\frac{d^2x}{d\tau^2} + \left[\theta_0^2 F(\tau) + \Delta\right]x = 0 \qquad (30)$$

Where $\theta_0^2 = \frac{q\beta G\lambda^2}{\gamma mc}$ -- dimensionless quadrupole strength,

$\Delta = \frac{\pi q E_0 T\lambda \sin\phi}{\gamma^3 mc^2\beta}$ -- dimensionless RF defocusing force.

$\tau = z/\beta\lambda$

$F(\tau) = 1, 0,$ or -1 -- periodic function

Matrix notation is more convenient.

Thin lens approximation of RF defocusing:

$$\frac{1}{f_g} = \frac{\Delta x'}{x} = \frac{\pi q E_0 T \sin(-\phi)}{\beta^2 \gamma^3 mc^2}$$

The transfer matrix through a period:

$$P = F_{1/2} L G L D L G L F_{1/2}$$

$$F_{1/2} = \begin{bmatrix} \cos(\kappa l/2) & \frac{1}{\kappa}\sin(\kappa l/2) \\ -\kappa \sin(\kappa l/2) & \cos(\kappa l/2) \end{bmatrix}$$

$$D_{1/2} = \begin{bmatrix} \cosh(\kappa l) & \frac{1}{\kappa}\sinh(\kappa l) \\ \kappa \sinh(\kappa l) & \cosh(\kappa l) \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0 \\ \frac{1}{f_g} & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

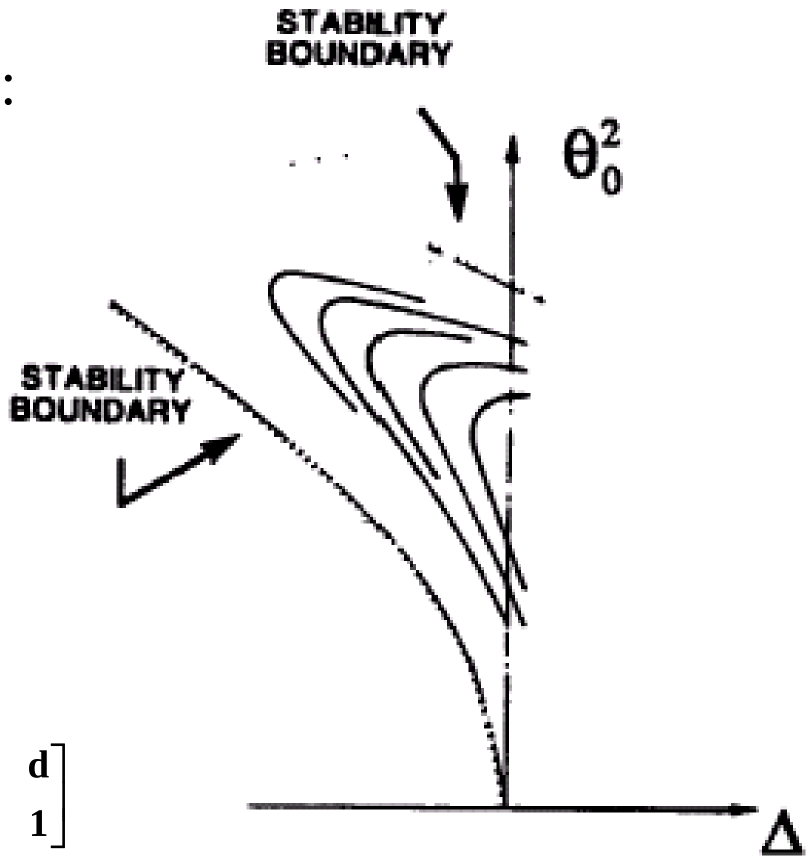


Fig. 15

Stability condition: $\mathrm{Tr}|P| = |P_{11} + P_{22}| \leqslant 2$

and phase advance in one period:

$$\sigma^2 \cong \left[\frac{qG\ell L}{mc\gamma\beta}\right]^2 - \frac{4\pi q E_0 T \sin(-\phi) L^2}{mc^2 \lambda (\gamma\beta)^3} \qquad (31)$$

3, Space charge effect – electromagnetic interaction between particles in a beam-repulsive

- High intensity linac has space charge effects which may result in

(1) A large beam radius(Fig.16);

(2) Beam emittance growth(Fig.17);

(3) Beam halo formation(Fig.18);

(4) Beam losses.

Usually it becomes limitation of beam current for high intensity machine.

- In the moving frame with the beam, one particle sees only electrostatic repulsive force from other particles, which can be expressed by Coulomb formula:

$$\vec{E}_r = \frac{q\vec{r}}{4\pi\varepsilon_0 r^2}$$

While in the lab frame, the moving particle form currents which are attractive with each other, and thus the Lorentz force is:

$$F_r = q(E_r - vB_\theta) = q(1-\beta^2)E_r = qE_r/\gamma^2$$

(1) A large beam radius-linear space charge force

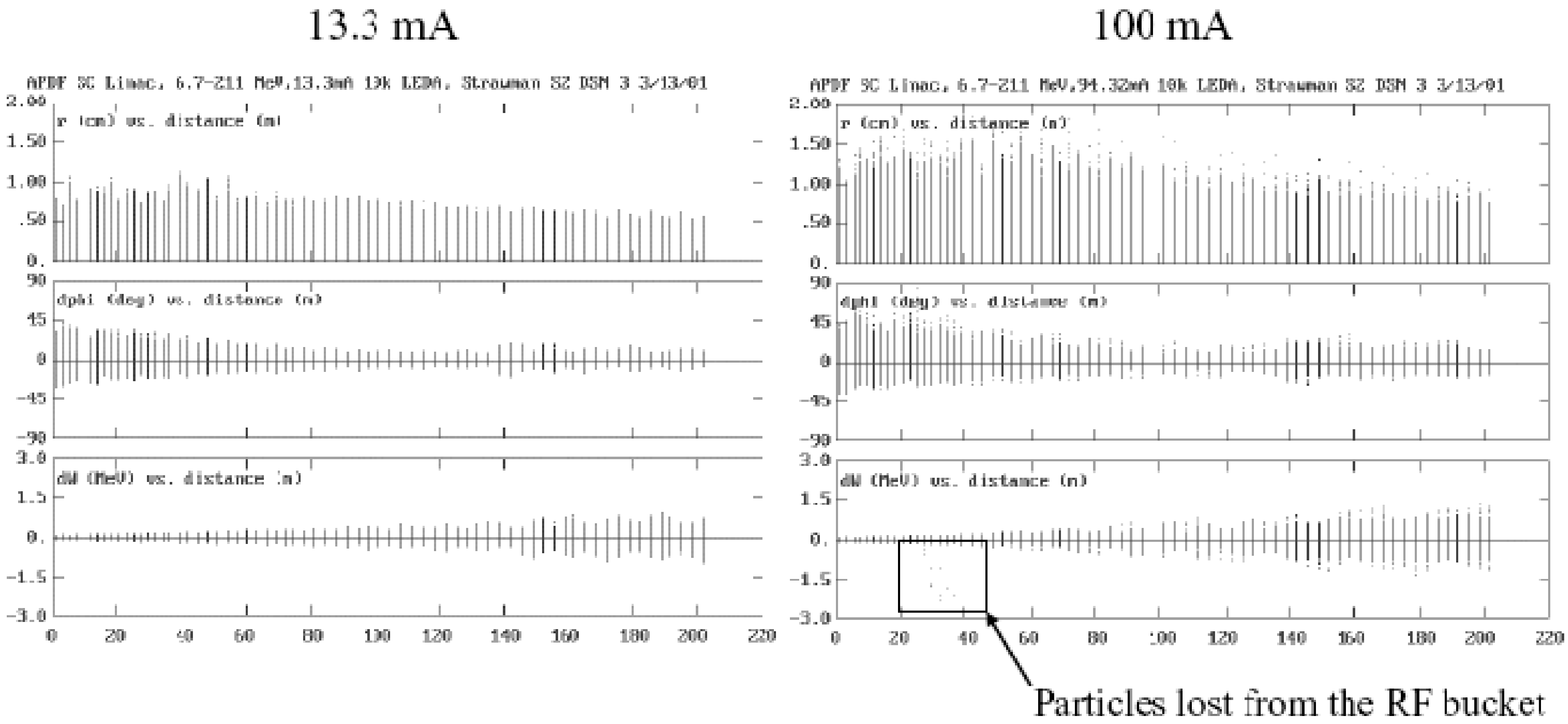


Fig. 16

(2) Beam emittance growth-nonlinear space charge force

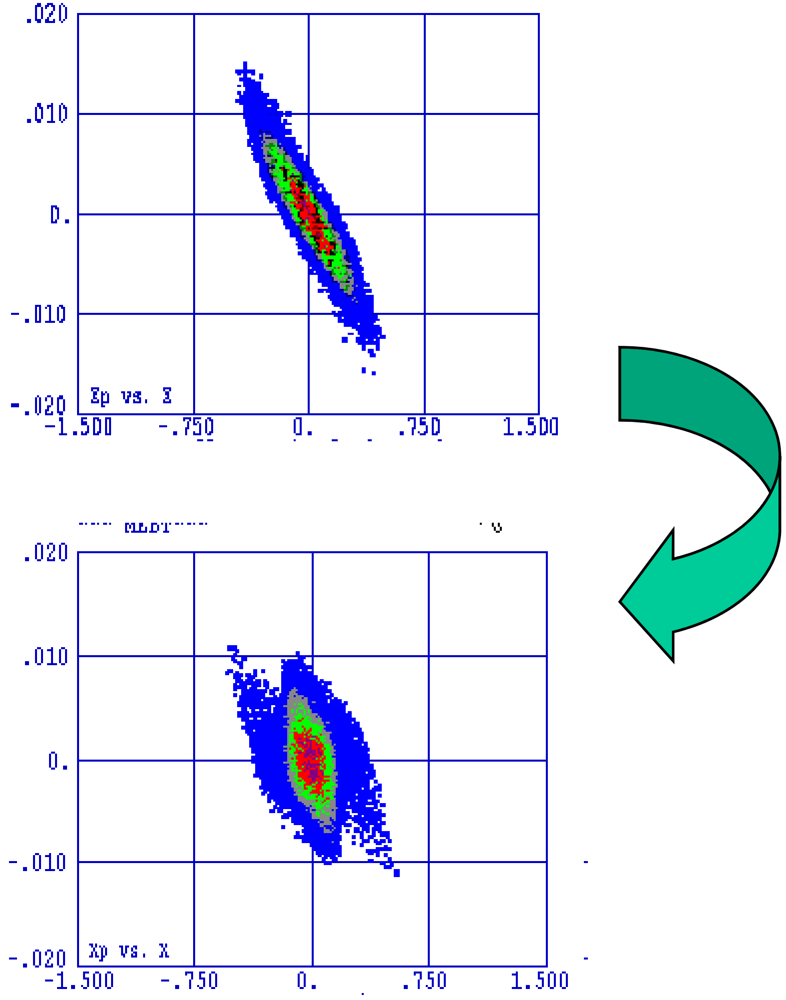


Fig.17

(3) Beam halo formation-nonlinear instability

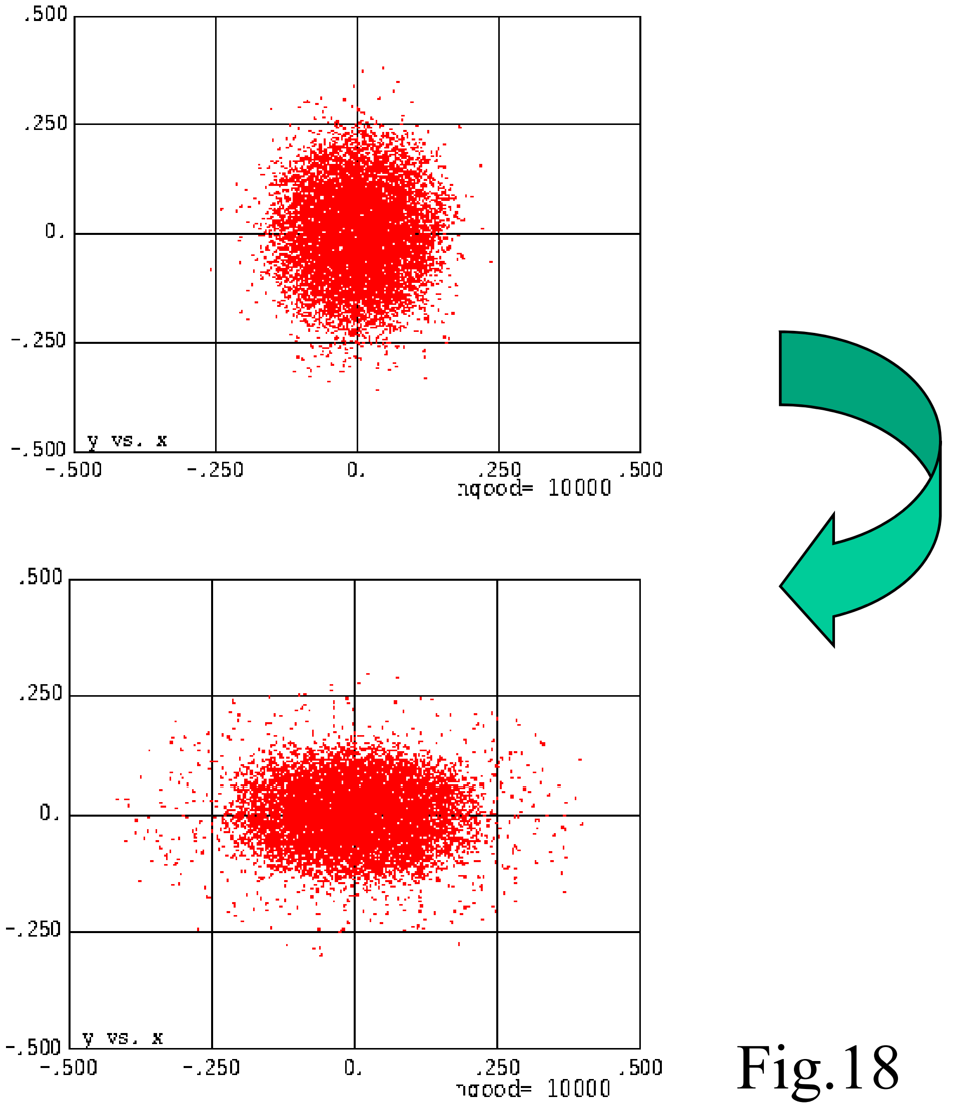


Fig.18

So the transversal equation of motion in x-direction is:

$$\frac{d^2 r}{dt^2} = \frac{qE_r}{\gamma^3 m}$$

The field is dependent on the particle distribution. For uniform cylindrical beam with of DC current, as a simple example:

$$E_r = \frac{I}{2\pi\varepsilon_0 c\beta R^2} r$$

with $r<R$, the beam radius R. In this case the transversal motion equation of a particle in a linac becomes:

$$\frac{d^2 r}{dz^2} + \kappa^2(z) r - \frac{k_{l0}^2}{2} r - \frac{K}{R^2} r = 0$$

with $K = \dfrac{qI}{2\pi\varepsilon_0 mc^3\beta^3\gamma^3}$, called as generalized perveance.

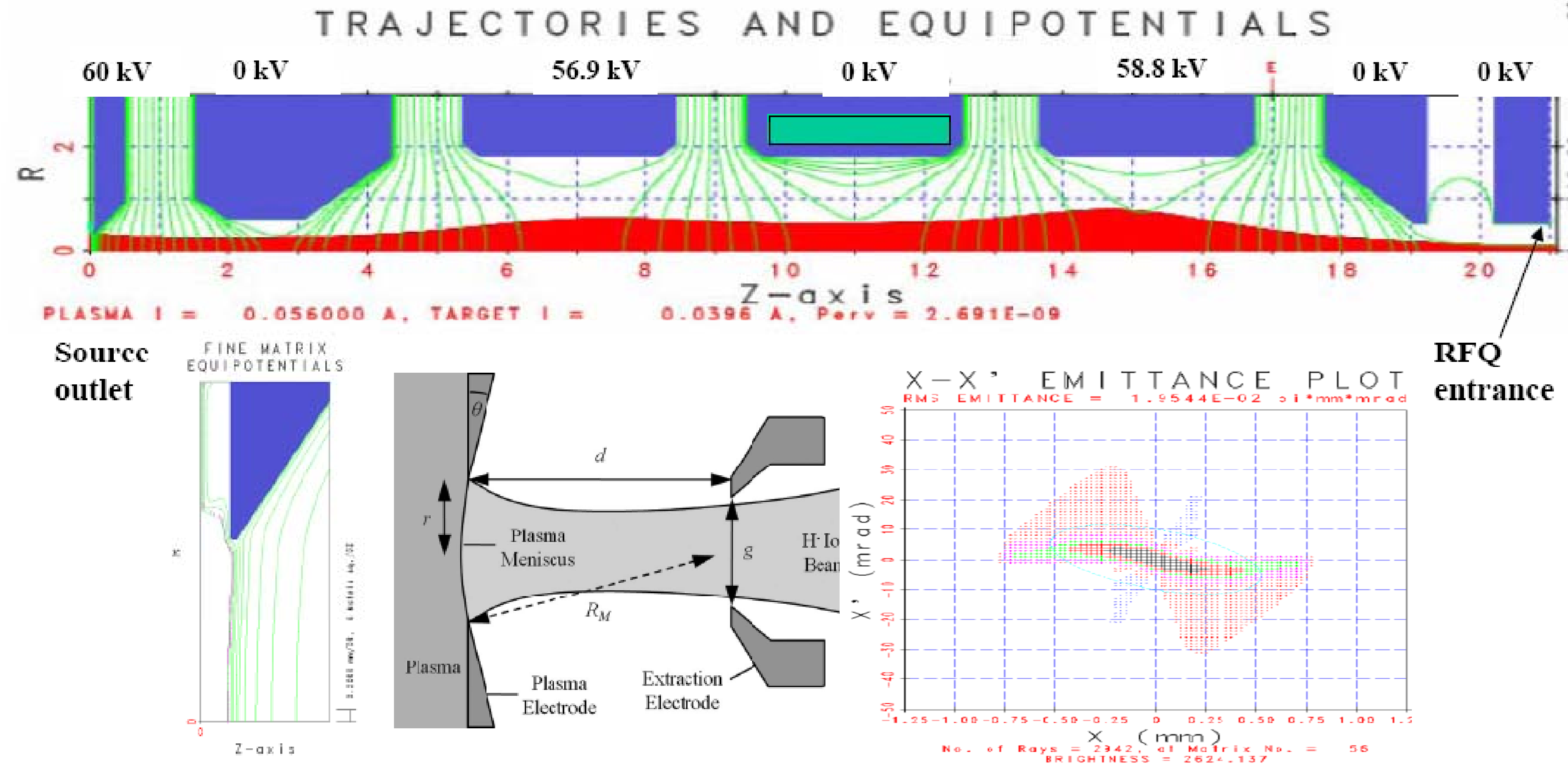
For 3D uniform ellipsoidal distribution in bunched beam,the field is:

$$E_{sx} = \frac{3I\lambda(1-f)}{4\pi\varepsilon_0 c(r_x + r_y) r_z} \frac{x}{r_x}, \quad E_{sy} = \frac{3I\lambda(1-f)}{4\pi\varepsilon_0 c(r_x + r_y) r_z} \frac{y}{r_y}, \quad E_{sz} = \frac{3I\lambda f}{4\pi\varepsilon_0 c r_x r_y} \frac{z}{r_z},$$

with f: form factor, r_x,r_y and r_z: the beam radius in x,y and z.

4, Dynamics codes (examples)

PBGun/IGUN: ion extraction simulation from a plasma with electrodes. Electrostatic field from electrodes and space charge can be generated on meshes by numerically solving Poisson equation. Ion rays are generated from the plasma and then traced through electrode system of LEBT.

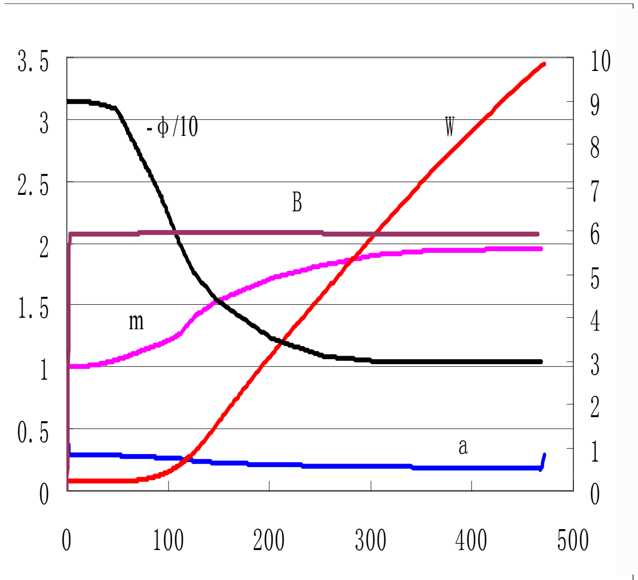


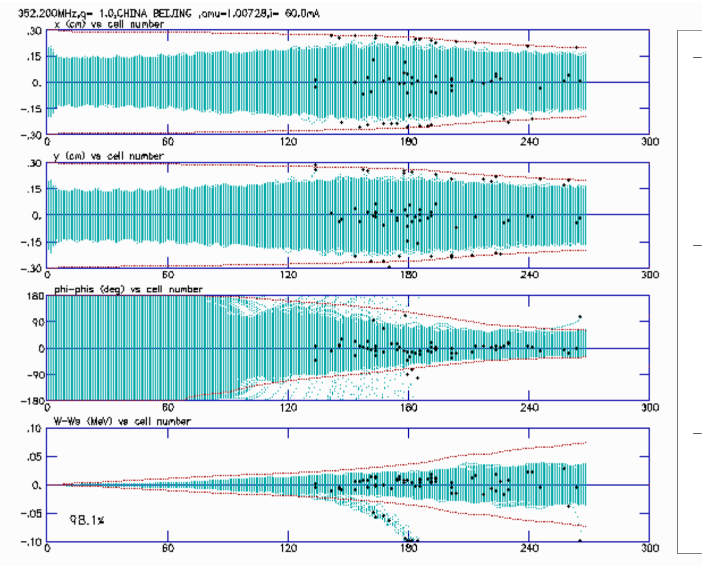
PARMTEQ: beam dynamics design and multi-particle simulation for Radio-Frequency Quadrupole linac(RFQ), with 2D space charge effect (in PIC model).

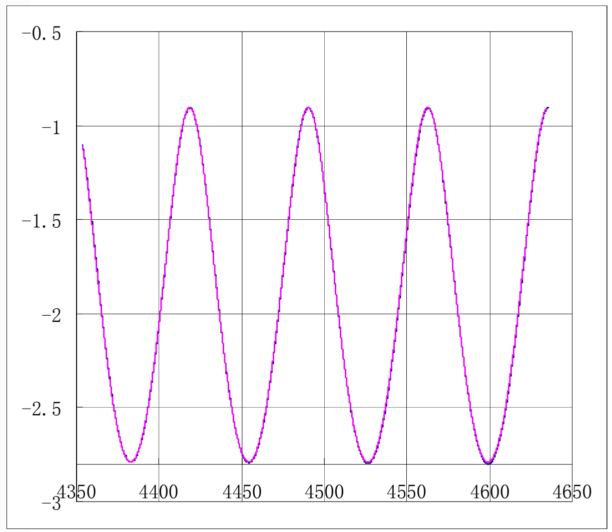
Parameters design with CURLI and RFQUIK codes

Cell design and multi particle tracking with space charge, with PARMTEQ code

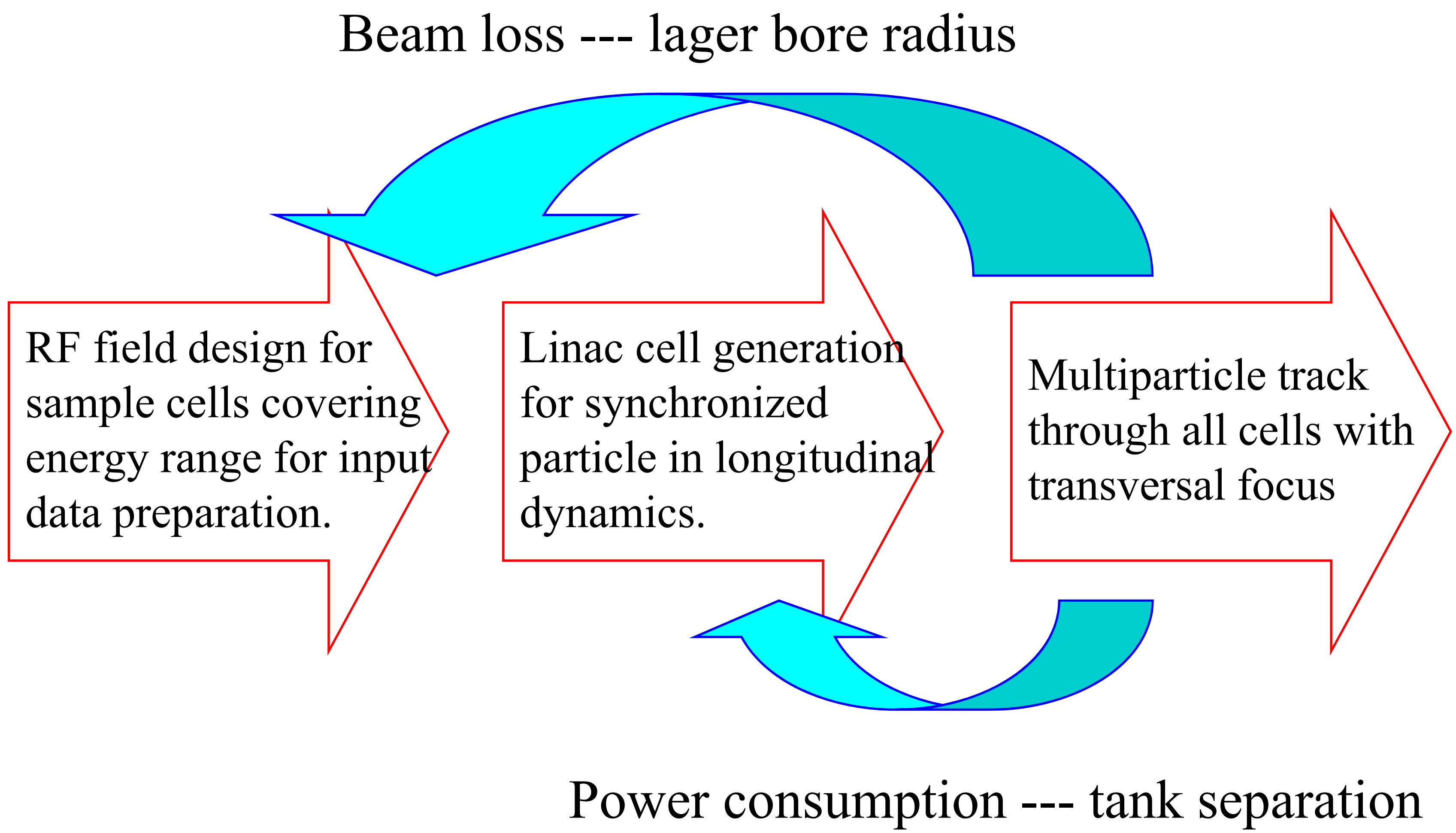
Manufacture vane curve generation with VANES code





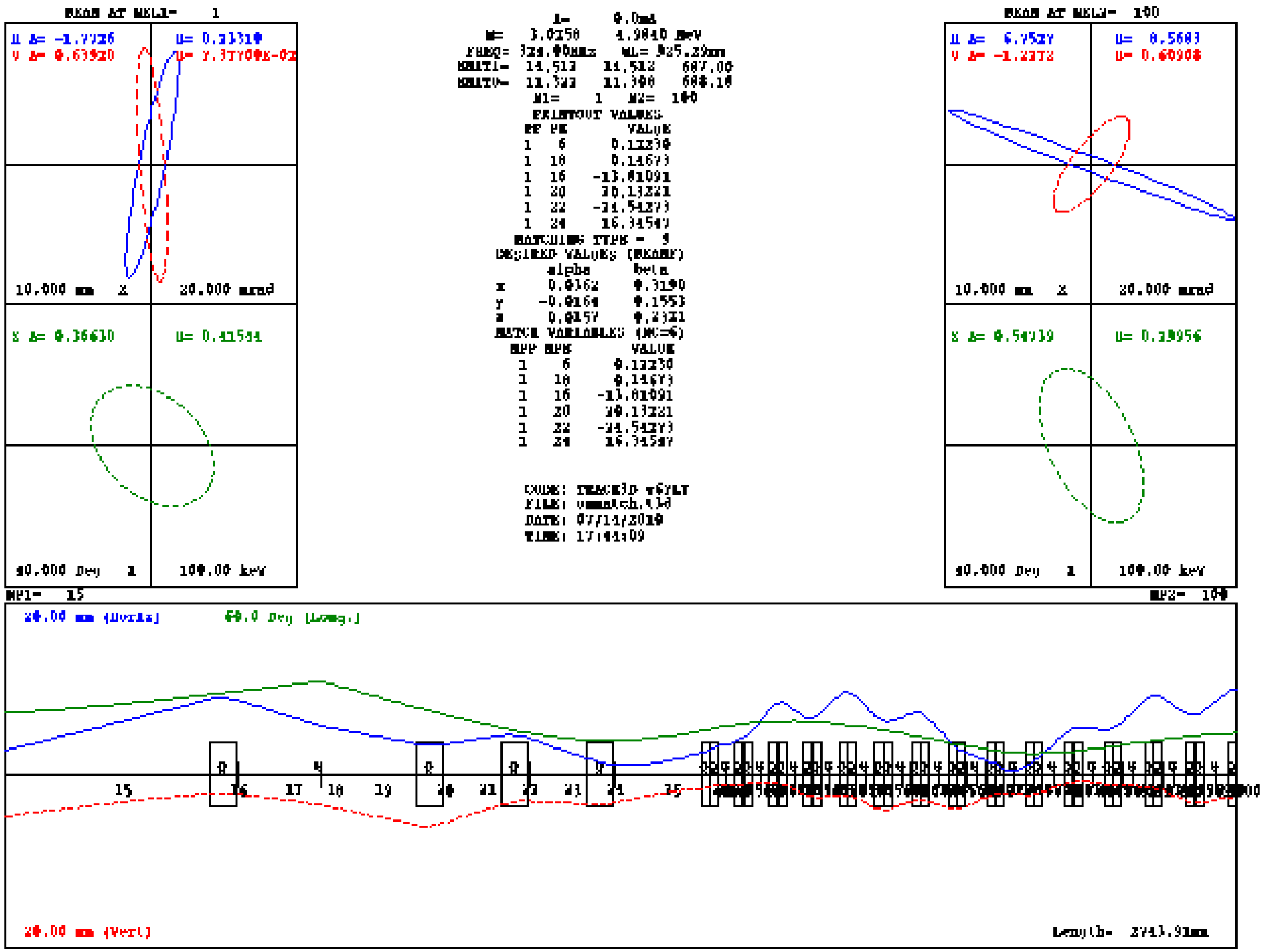


PARMILA: an ion beam-dynamics code that performs two tasks: generates an SW linac and then transports particles through the linac with 2D space charge effect (in PIC model).



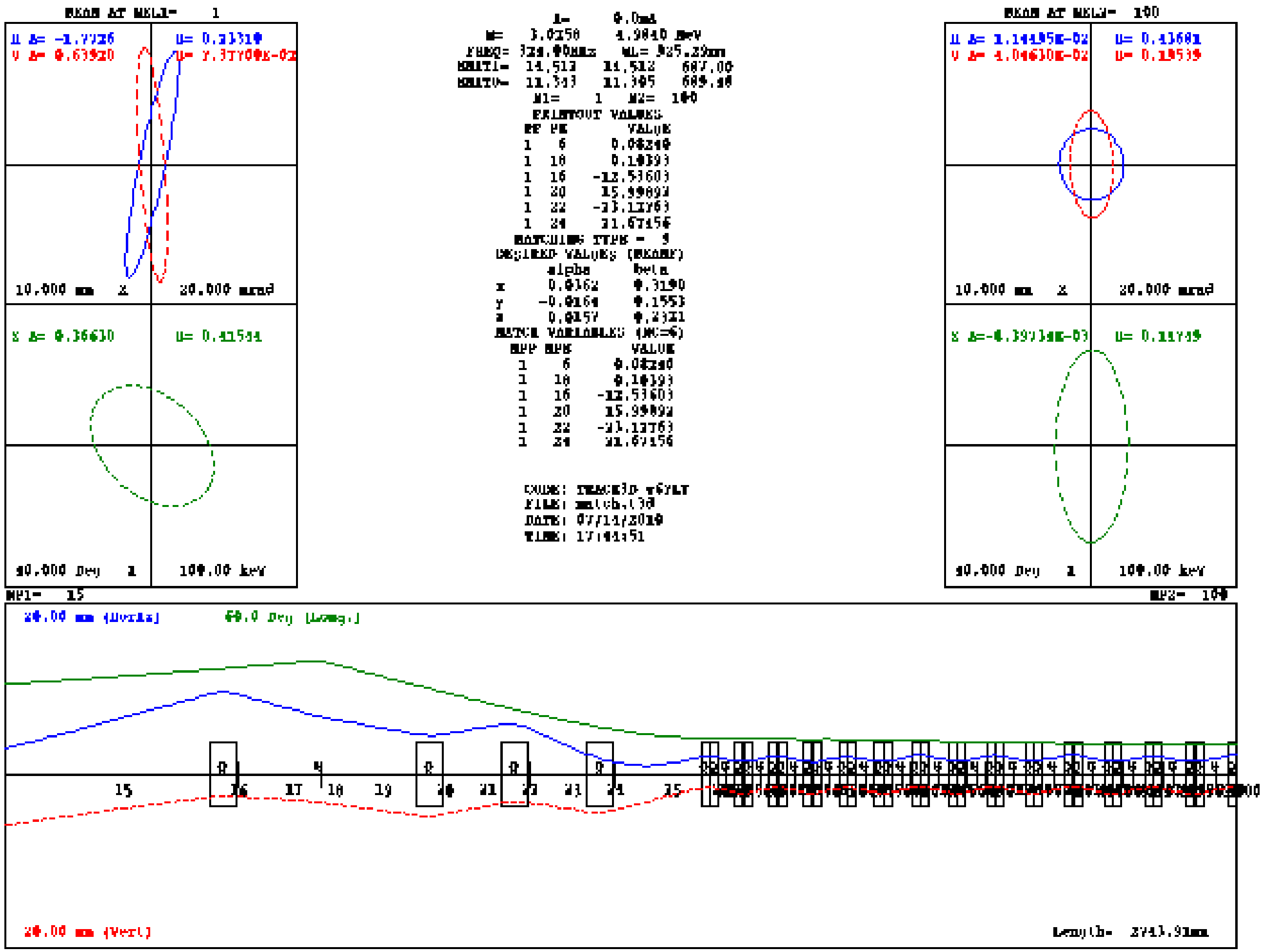
TRACE3-D: an interactive beam-dynamics program that calculates the envelopes of a bunched beam, including linear space-change forces, through a user-defined transport system.

Perform beam match function between different accelerating structures.



TRACE3-D: an interactive beam-dynamics program that calculates the envelopes of a bunched beam, including linear space-change forces, through a user-defined transport system.

Perform beam match function between different accelerating structures.



Summery

1, Synchronism condition: L=vT=βλ for 0 mode (or βλ/2 for π mode)

2, Energy gain in RF field: $\delta W = qE_0 TL\cos\phi$

3, Transit-time factor:

$$T = \frac{\int_{-L/2}^{L/2} E(0,z)\cos(2\pi z/\beta\lambda)dz}{\int_{-L/2}^{L/2} E(0,z)dz}$$

4, Longitudinal motion

$$\frac{d}{dz}\left[\beta_s^3\gamma_s^3\frac{d}{dz}\Delta\phi\right] = -\frac{2\pi}{\lambda}\frac{qE_m}{mc^2}(\cos\phi - \cos\phi_s)$$

and stability $-2|\phi_s| < \Delta\phi < |\phi_s|,\ \phi_s < 0$

5, Transverse motion

$$\frac{d^2x}{dz^2} + \kappa^2(z)x - \frac{k_{l0}^2}{2}x - \frac{K}{R^2}x = 0$$

and stability $Tr|P| \leqslant 2$

Linac Structure

Contents

1. Slow-wave structure
2. Figures of merit of a structure
3. Standard linac structures
4. Design codes of linac structures

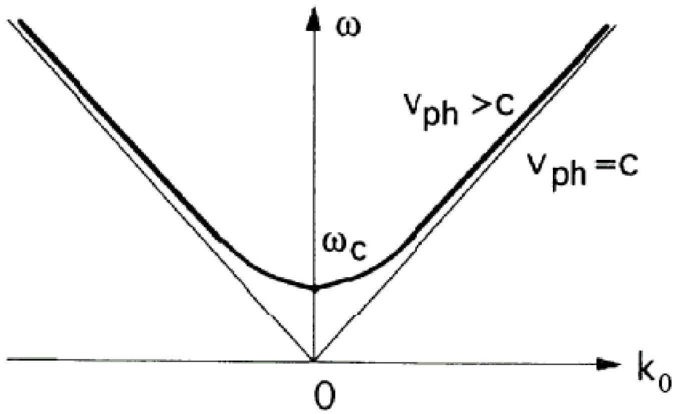
1, Slow-wave structure

Problem:synchronism condition requires v_p=v, but in a uniform cylinder wave guide $v_p > c$, according to the dispersion relation:

$$\omega^2 = (Kc)^2 + (k_0 c)^2$$

where K is the cutoff wave-number for the TM_{01} mode.

$$v_p \equiv \frac{\omega}{k_0} \qquad v_p = \frac{c}{\sqrt{1-(Kc)^2/\omega^2}} > c$$



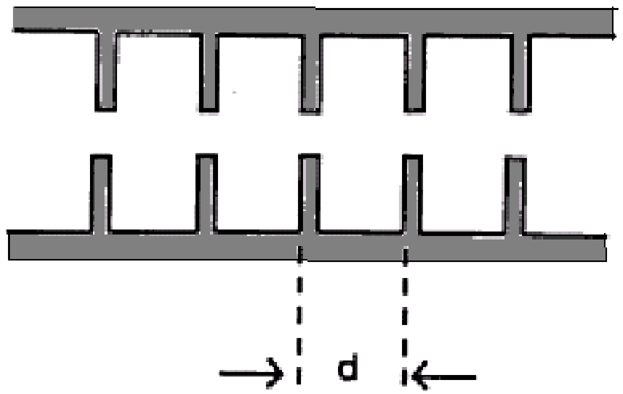
Solution: periodically loaded wave-guide

Floquet Theorem: one period difference of field is exp(jk_0d).

$$E_z(r,z,t) = E_d(r,z)e^{j(\omega t - k_0 z)}$$

where $E_d(r,z)$ is a periodic function of d, and can be expended in Fourier series:

$$E_d(r,z) = \sum_{n=-\infty}^{\infty} a_n(r) e^{-j2\pi nz/d}$$



$$a_n(r) = E_n J_0(K_n r)$$

$$K_n^2 = (\omega / c)^2 - (k_0 + 2\pi n / d)^2$$

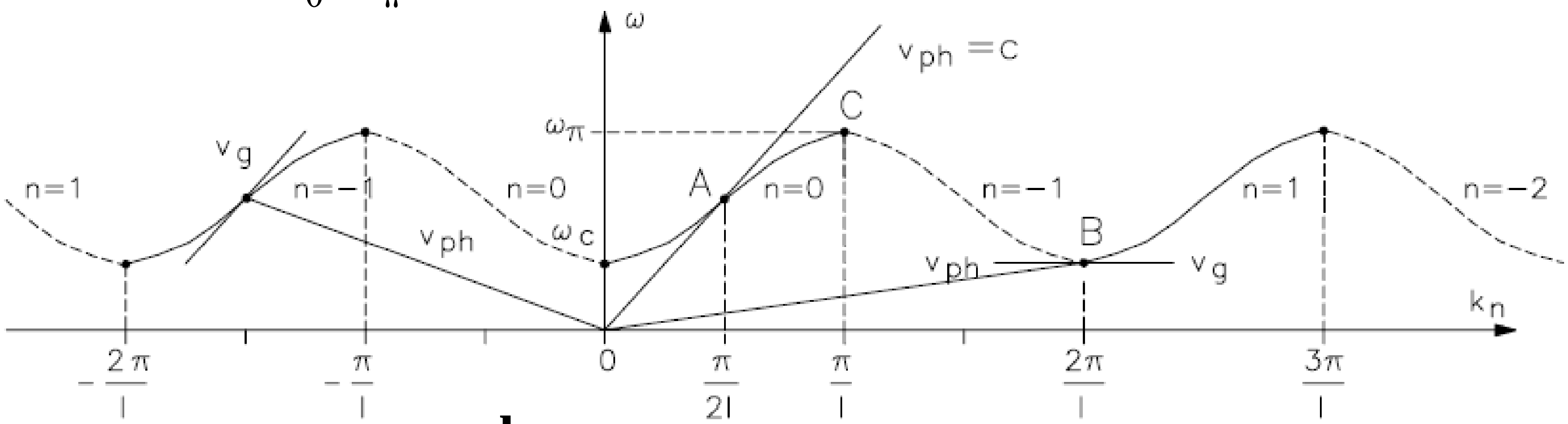
$$E_z(r,z,t) = \sum_{n=-\infty}^{\infty} E_n J_0(K_n r) e^{j(\omega t - k_n z)} .$$ space harmonics

We define the wave-number for the nth space harmonic as $k_n = k_0 + \frac{2\pi n}{d}$

The phase velocity for the nth space harmonic is

$$\beta_n = \frac{\omega}{k_n c} = \frac{\beta_0}{1 + (n\beta_0 \lambda / d)}$$

Pass band: ω_0-ω_π

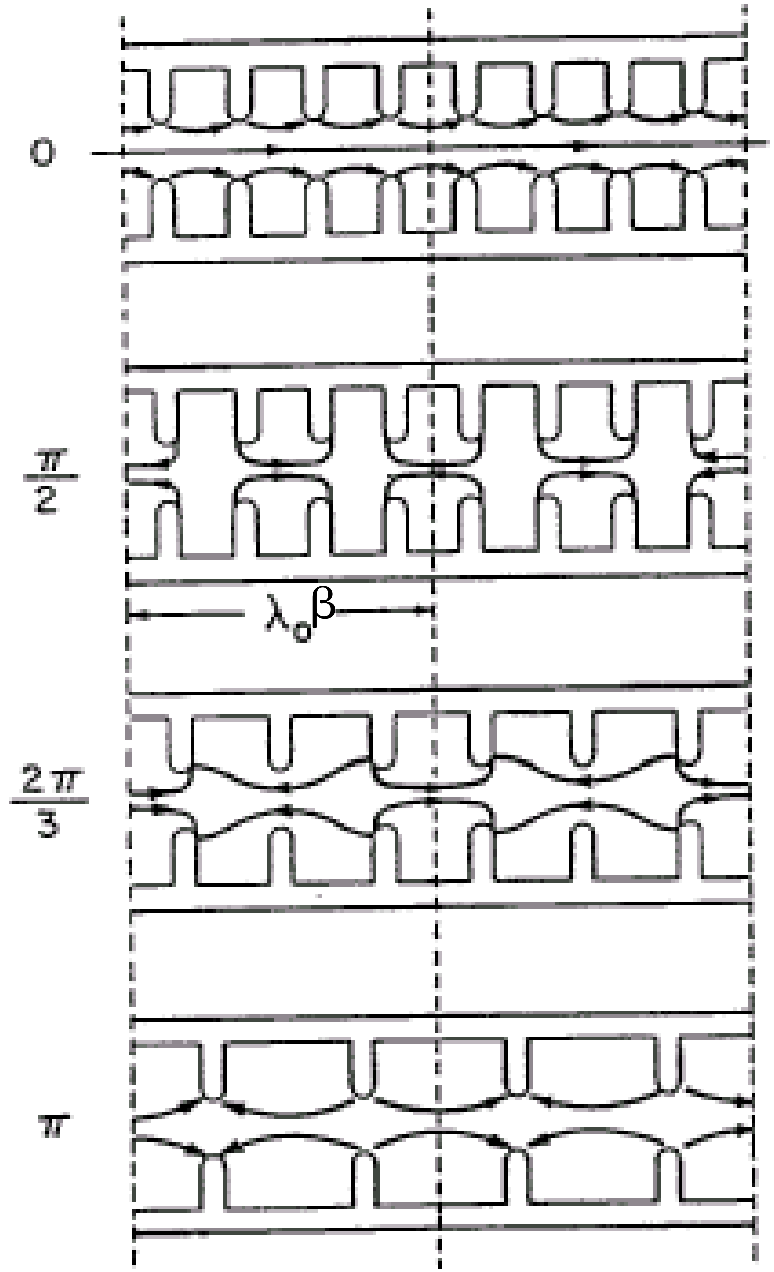


Group velocity $v_g = \frac{d\omega}{dk}$ .It becomes zero at the ends of the pass band.

Structure mode for n=0 harmonic:

cell-to-cell phase shift

$k_0d=0,\pi/2,2\pi/3,\pi$



2, Figures of merit of a structure

1) Quality factor: $Q=\omega U/P$

2) Shunt impedance: $r_s \equiv V_0^2 / P$

Peak energy gain: $\Delta W_{\phi=0} = qE_0LT = q\sqrt{(r_s T^2)P}$.

Effective shunt impedance:

$$r \equiv r_s T^2 = \left[\frac{\Delta W_{\phi=0}}{q}\right]^2 \frac{1}{P} = \frac{[E_0 TL]^2}{P}$$

Shunt impedance per unit length:

$$Z \equiv \frac{r_s}{L} = \frac{E_0^2}{P/L} \qquad ZT^2 = \frac{(E_0 T)^2}{P/L} \ [\mathrm{M\Omega m^{-1}}]$$

3) r/Q:

$$\frac{r}{Q} = \frac{(V_0 T)^2}{\omega U}$$

4) RF power efficiency

$\eta=P_b/P_T$, with $P_b=I\bullet\Delta W/q$, $P_T=P+P_b$

Scaling with RF frequency

Let E_0 fixed, ΔW fixed, so $L \propto f^0$, and thus $T \propto f^0$, $E \propto f^0$, $B \propto f^0$,.

$S \propto f^{-1}$, $V \propto f^{-2}$. Consequently, we have:

$$R_s = \frac{1}{\sigma\delta}, \delta = \sqrt{\frac{2}{\sigma\mu_0\omega}}$$

$$R_s \propto \begin{Bmatrix} f^{1/2} & \text{normal conducting} \\ f^2 & \text{superconducting} \end{Bmatrix}$$

$$P = \frac{R_S}{2}\left|\frac{B}{\mu_0}\right|^2 dA \propto \begin{Bmatrix} f^{-1/2} & \text{normal conducting} \\ f & \text{superconducting} \end{Bmatrix}$$

$$Q = \frac{\omega U}{P} \propto \begin{Bmatrix} f^{-1/2} & \text{normal conducting} \\ f^{-2} & \text{superconducting} \end{Bmatrix}$$

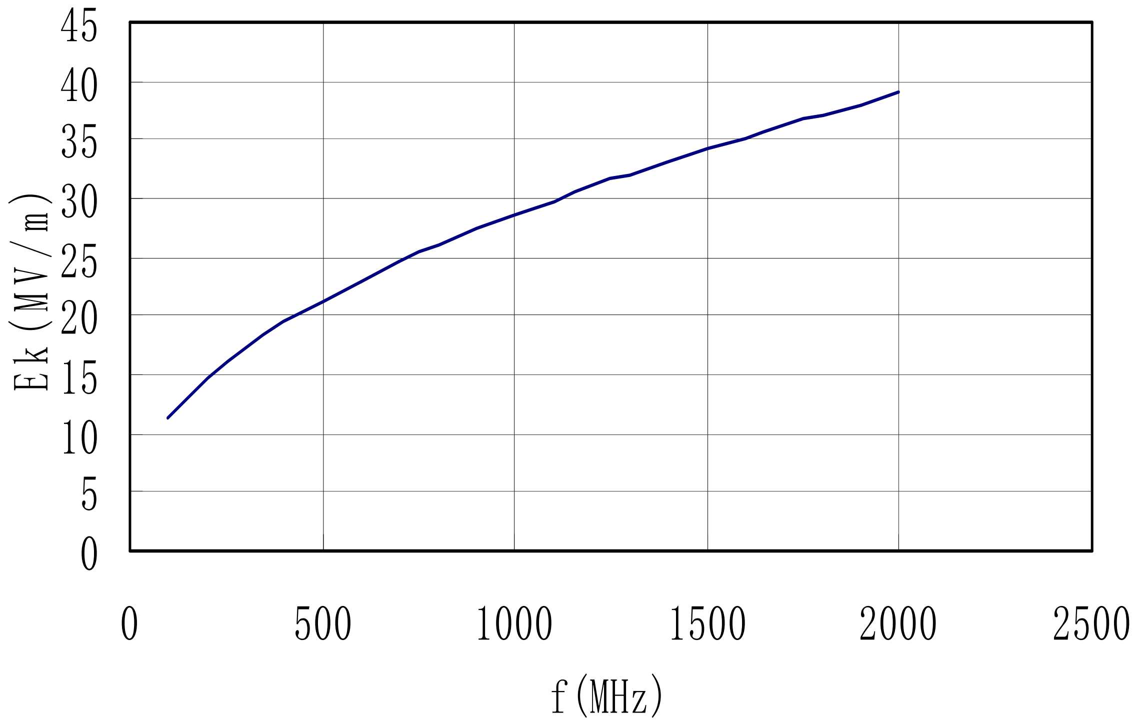
$$ZT^2 = \frac{E_0 T^2 L}{P} \propto \begin{Bmatrix} f^{-1/2} & \text{normal conducting} \\ f^{-1} & \text{superconducting} \end{Bmatrix}$$

$$\frac{ZT^2}{Q} \propto \begin{Bmatrix} f^1 & \text{normal conducting} \\ f^1 & \text{superconducting} \end{Bmatrix}.$$

Kilpatrick field limit is also related with frequency:

$$f = 1.643 E_k^2 \exp\left(\frac{-8.5}{E_k}\right)$$

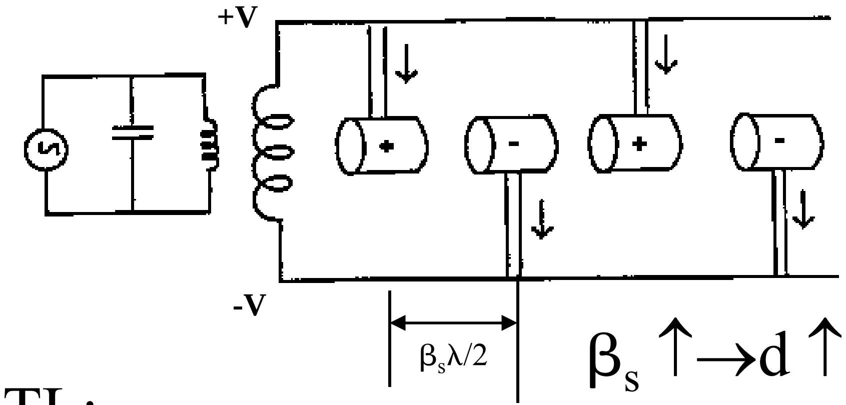
Ek: the maximum surface field.



Design field levels for modern accelerating cavities are typically in the range from $1.0E_k$ to $2.0E_k$.

3, Standard linac structures

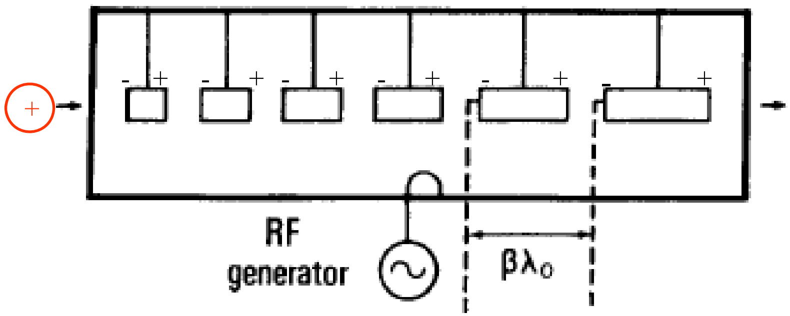
1) Wideroe linac: a π-mode SW structure. At 20-100 MHz



$\beta_s \uparrow \rightarrow d \uparrow \rightarrow E \downarrow$ (V=const)

2) Alvarez DTL:

a 0-mode SW structure. At 200-400MHz

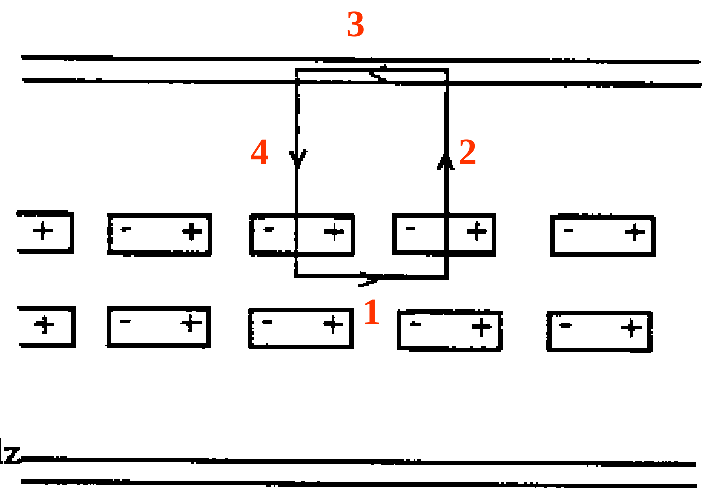


Faraday's Law: $\oint \vec{E} \bullet d\vec{\ell} = -\frac{\partial}{\partial t}\int \vec{B} \bullet d\vec{A}.$

$$\int_1 \vec{E} \bullet d\vec{\ell} = E_0\beta\lambda \qquad -\frac{\partial}{\partial t}\int \vec{B} \bullet d\vec{A} = -j\omega\Phi$$

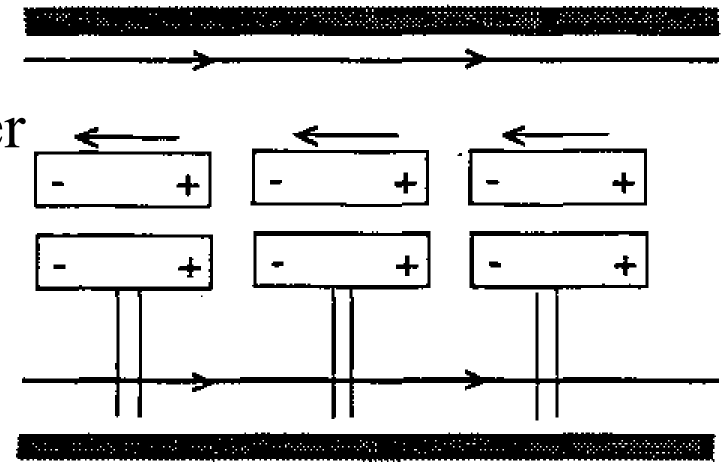
$$\Phi = \int B_\theta \, dr dz$$

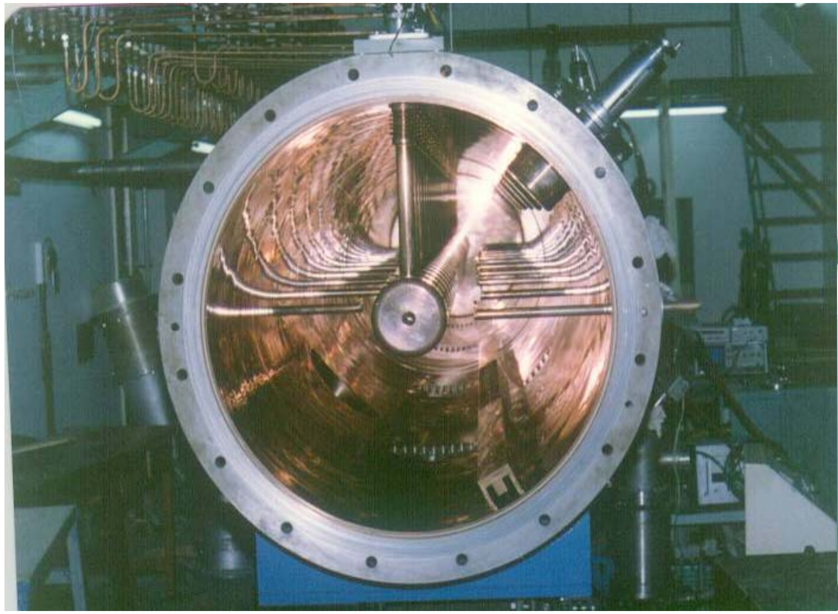
We get: $E_0 = -\frac{j\omega\Phi}{\beta\lambda}$

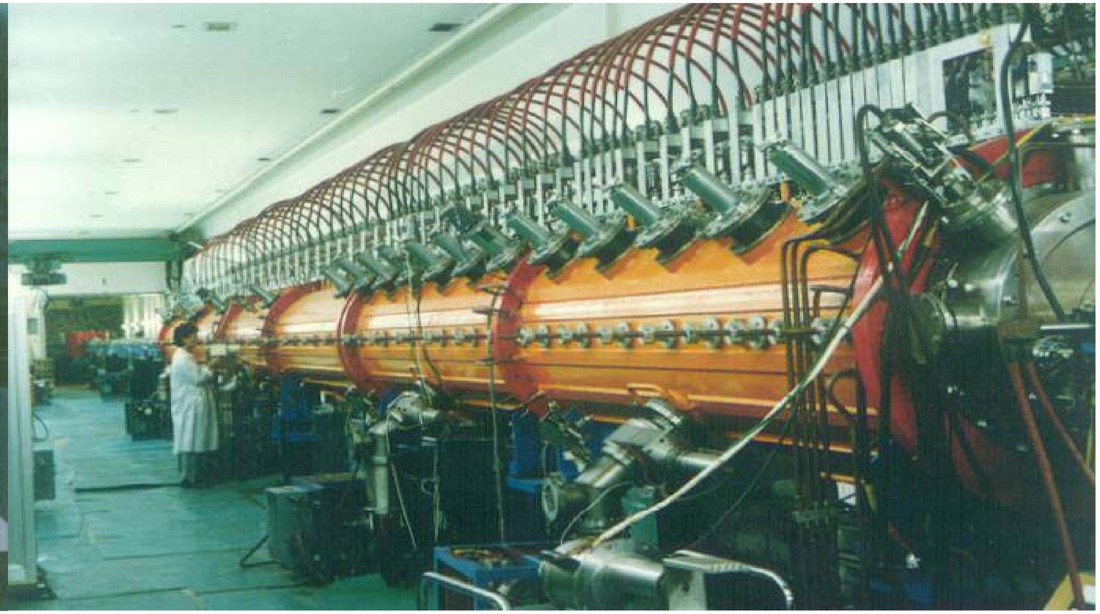


Observation: E_0 is a constant if magnetic flux per unit length is a constant.

The currents flow longitudinally on the outer walls and on the DTs, and everywhere in phase. Conducting current on DTs and displacement current between DTs.







Summary of DTL

Applicable to proton or ion with $0.05<\beta<0.4$

Advantages:

- High ZT^2 due to open cell end
- Strong focusing with Q-magnet in DT

Disadvantages:

- T& ZT^2 decreases as β increases
- T& ZT^2 decreases at low β when aperture is fixed
- Focusing vanishes at low β (no room for Q)

3) Coupled-cavity linacs

For electrons and protons in the velocity range of about $0.3 < \beta < 1.0$

Side-coupled linac used at LAMPF, LANL

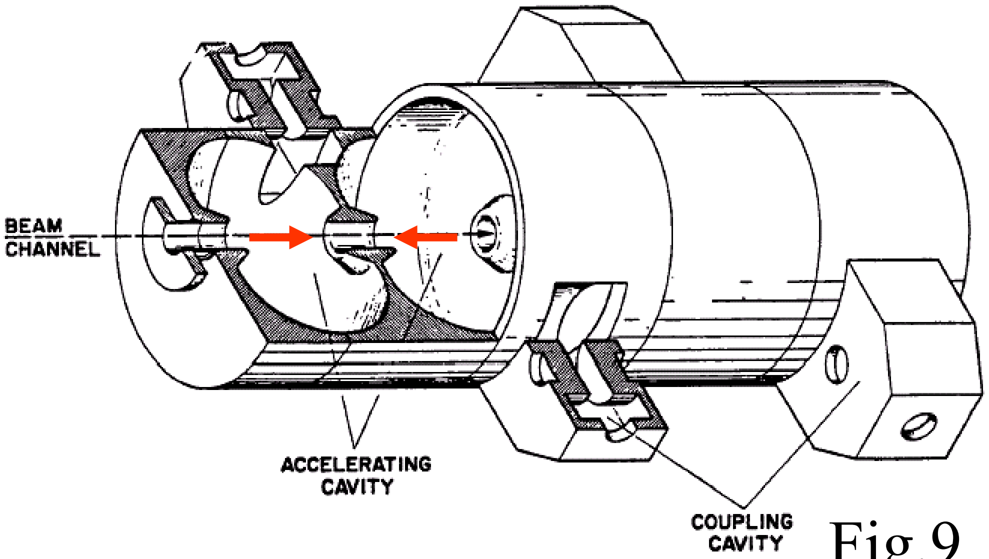
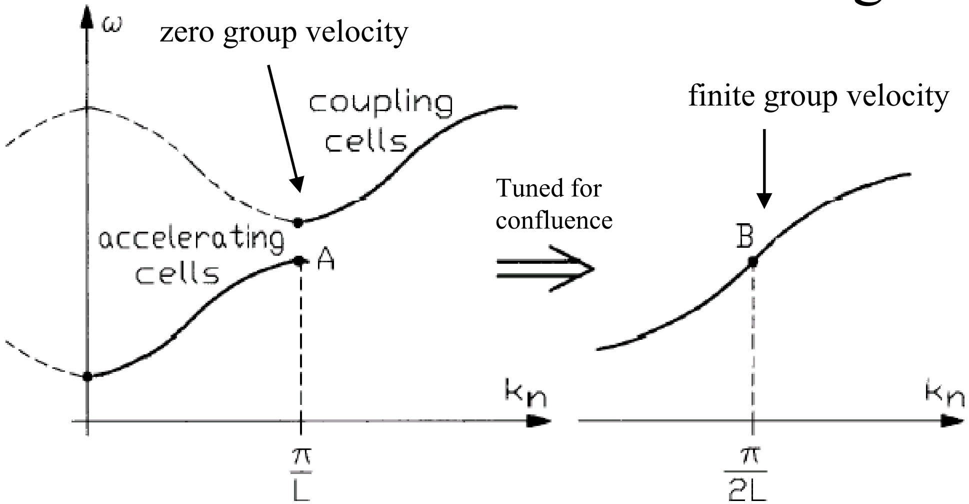


Fig.9

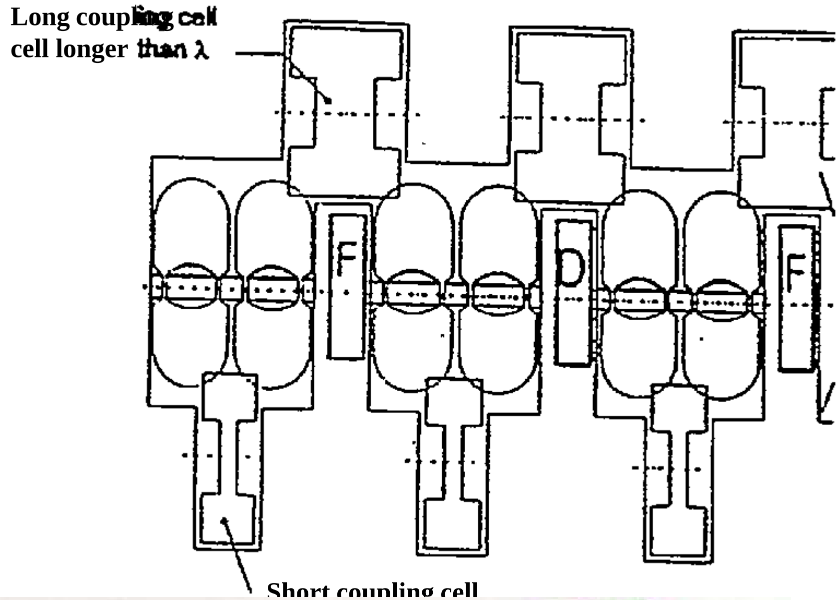
Operated in $\pi/2$ mode for cell-to-cell field pattern, but π mode for beam.

Biperiodic structures-turned for *confluence :*



So, stabilizing post is added in DTL to form a biperiodic structure.

4) Coupled-Cavity DTL (CCDTL)

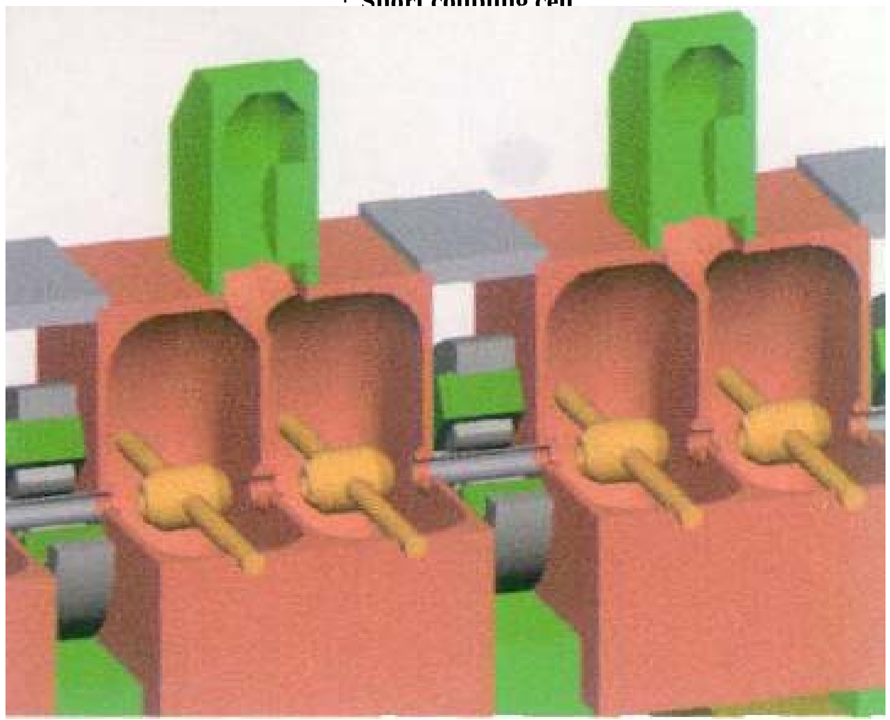


A combination of CCL and DTL structures

The accelerating cavity is a short 0-mode DTL

π/2 structure mode

E field is out of phase between adjacent cavities (π mode for beam).

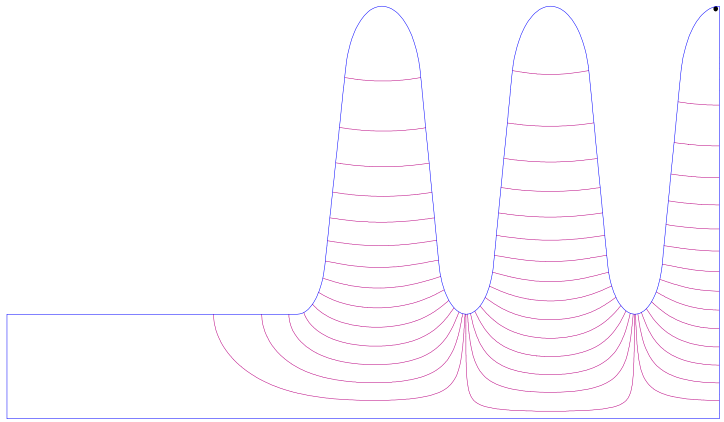


ZT^2 is higher than DTL

Q-magnet is out of the cavity

5) Superconducting structure

Why superconducting

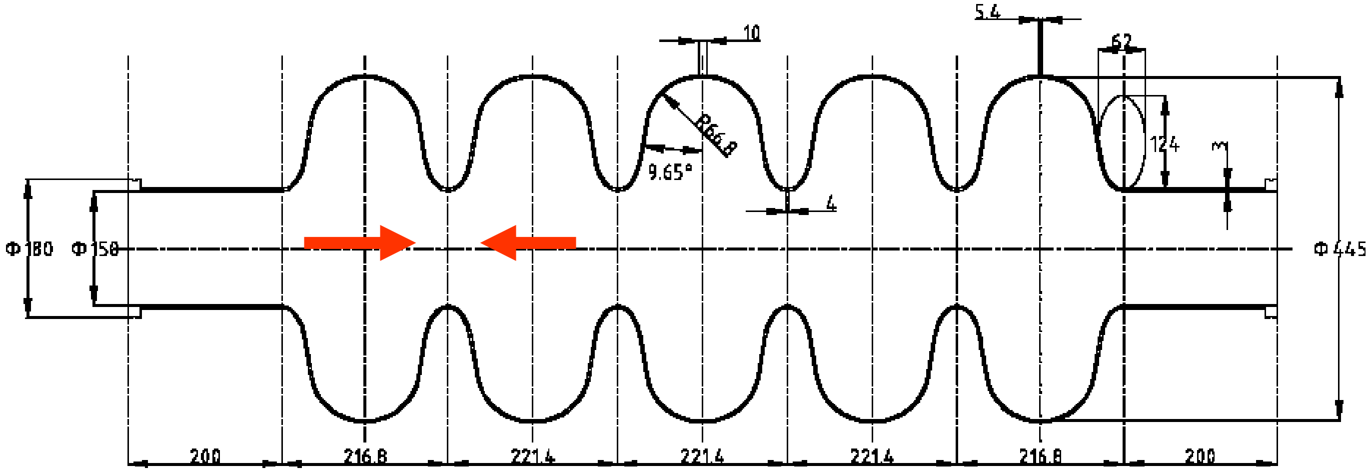


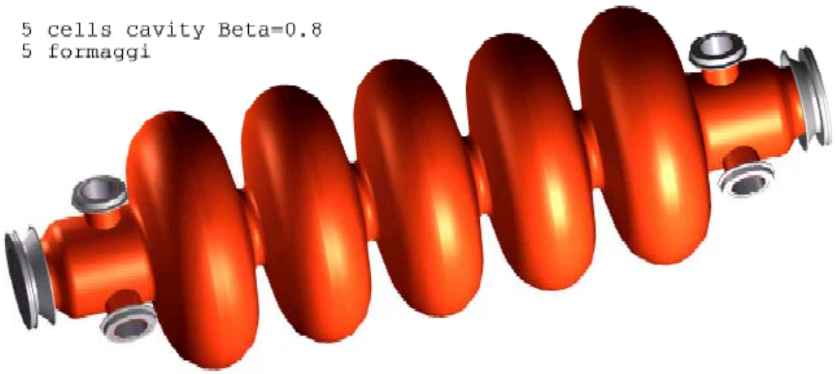
	SCL	Superconducting	
Frequency	425	352.2	MHz
Q	2.80E+04	2.00E+09	
beta	0.44	0.50	
Beam bore diameter	16.00	94.4	mm
r	38.00	2.22E+05	MΩ/m
r/Q	1357.	110.	Ω/m
I	30.	30	mA
E	2.	2	MV/m
η	**0.24**	**0.644**	

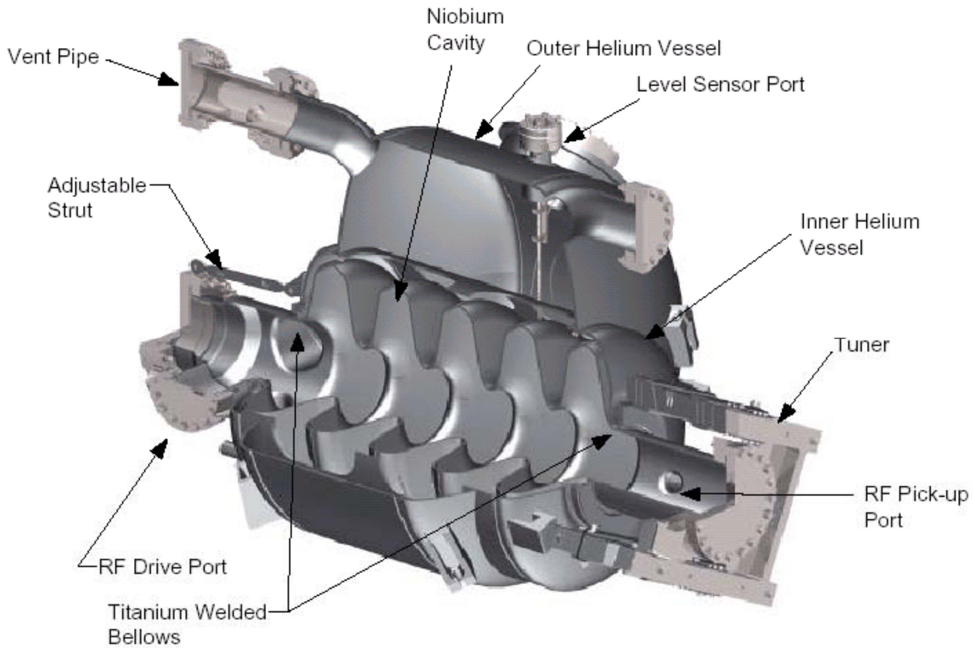
The static losses in the cryostats (to be added for the superconducting linac) can be dominating.

5) Superconducting structure

(1)Elliptic cavity







Elliptical cavity chain made of Niobium at 2-4.2 K. Working in π mode

5) Superconducting structure

Design Consideration：

1、**Esp/Eacc** 要小——防止场致发射、次级电子倍增；

2、**Bsp/Eacc** 要小——防止失超；

3、**Cell-to-cell coupling κ**大——场纵向稳定性**(1-2%)**；

4、束孔大———避免束流损失 **(**孔径比＝**20**－**30)**；

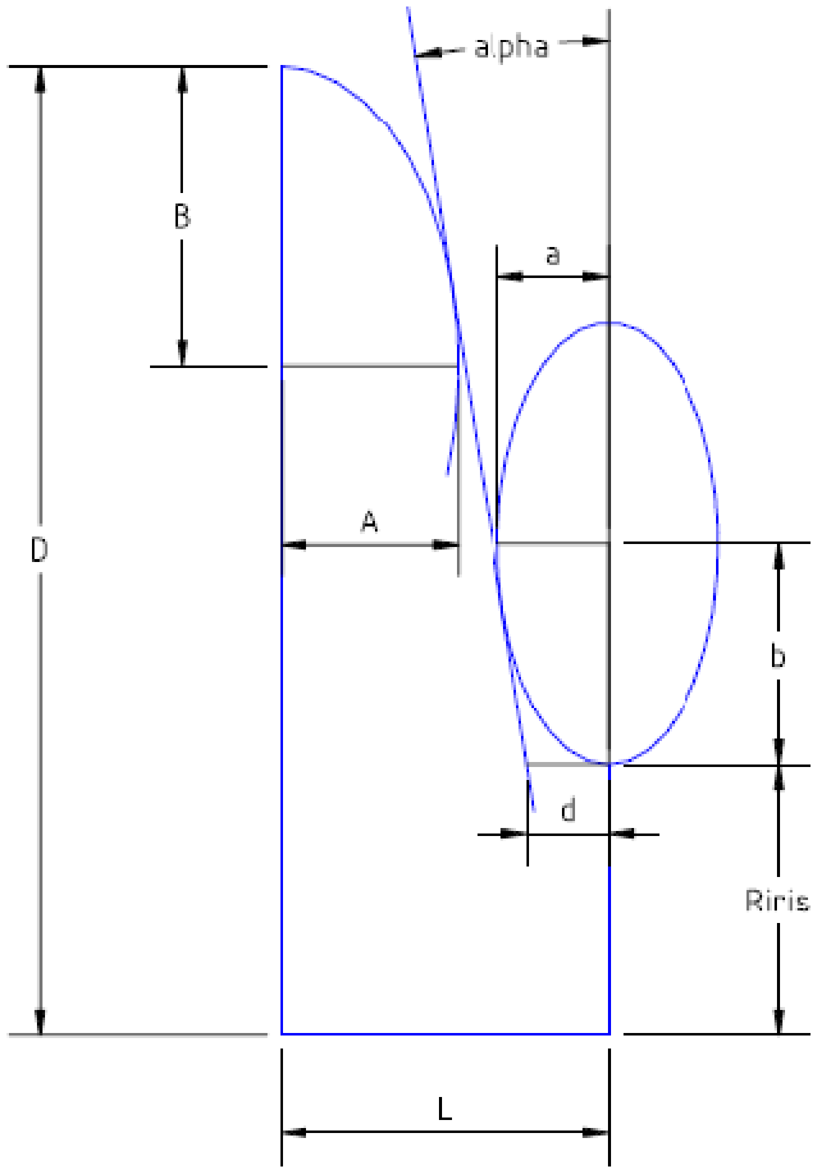
5、机械稳定性好——降低**Lorentz detuning factor K &** 升高**Microphonic** 频率.

(K--16Hz/(MV/m)^2 @ β=0.5)

5) Superconducting structure

Cell Geometrical Design

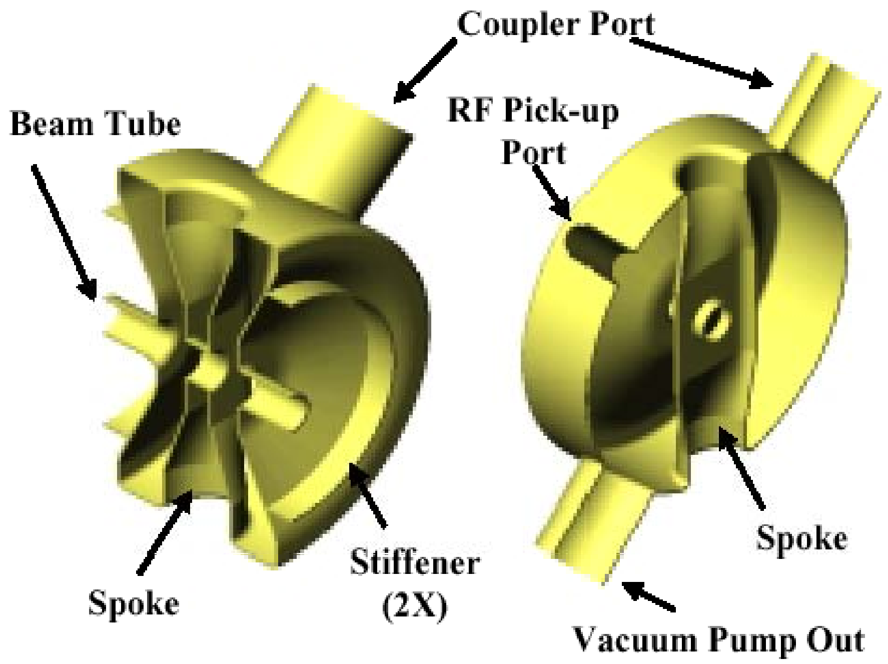
- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - ✓ Ellipse ratio at the equator (R=B/A)
    Ruled by Mechanics
  - ✓ Ellipse ratio at the iris (r=b/a)
    Epeak
  - ✓ Side wall inclination (α) and position (d)
    Epeak vs. Bpeak tradeoff and coupling k
  - ✓ Cavity iris radius R_{iris}
    Coupling k
  - ✓ Cavity Length L
    β
  - ✓ Cavity radius D
    used for frequency tuning
- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters



5) Superconducting structure

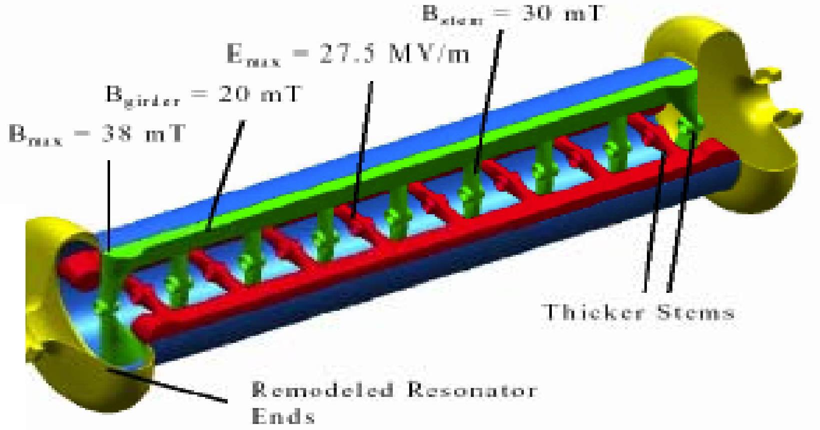
(2) Spoke cavity

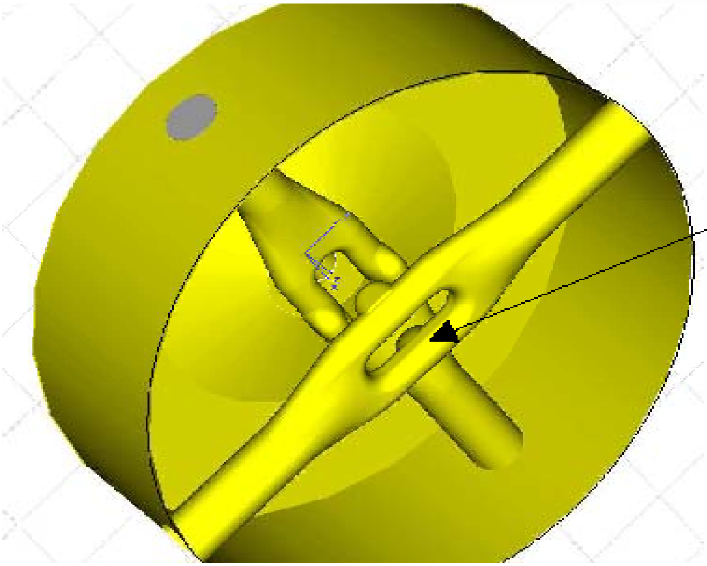
- 低β: 0.17
- 高度的机械稳定性。
- 尺寸紧凑，350 MHz的 Spoke 腔直径与 700 MHz 的椭圆超导腔差不多。
- 单元间存在强的磁耦合，因此可以采用较小的束流孔径。
- 容许较高的电场和磁场峰值。



5) Superconducting structure

Multi-cell spoke cavity and RF focusing spoke cavity

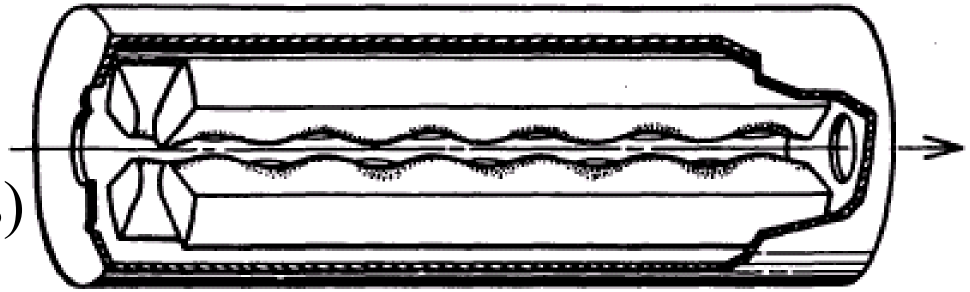




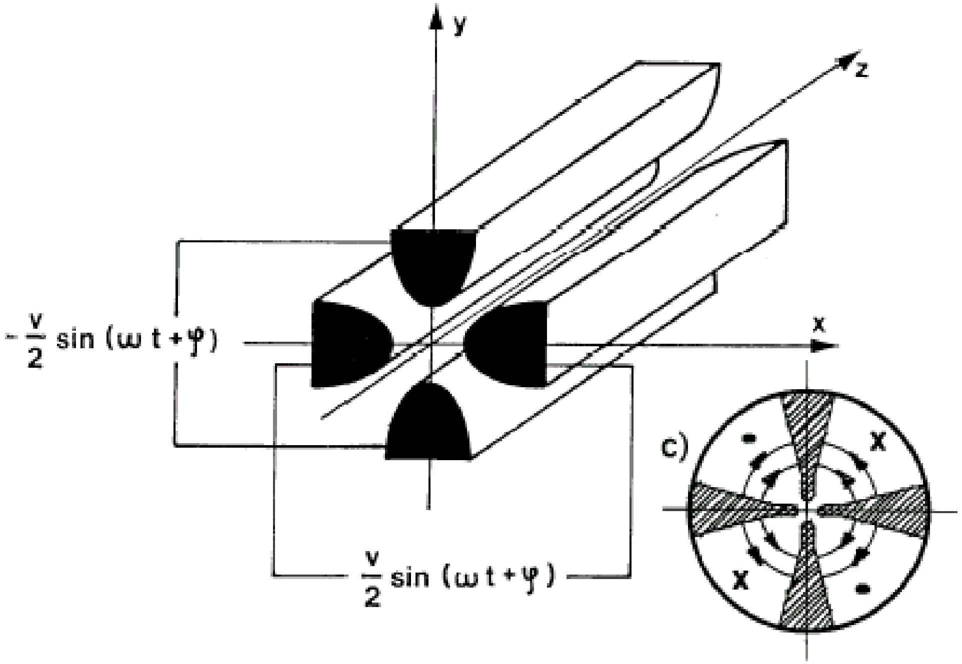
Elongated beam aperture reduces peak surface fields and cavity capacitance.

6) Radio-Frequency Quadrupole (RFQ)

The RFQ accelerator is a very particular ion linac because it accepts an unbunched (continuous) beam, bunches it, focuses it and accelerates it. It is the best one in the low energy range, from a few keV to a few MeV per atomic mass unit.



The operation principle of the RFQ is explained with the figures.

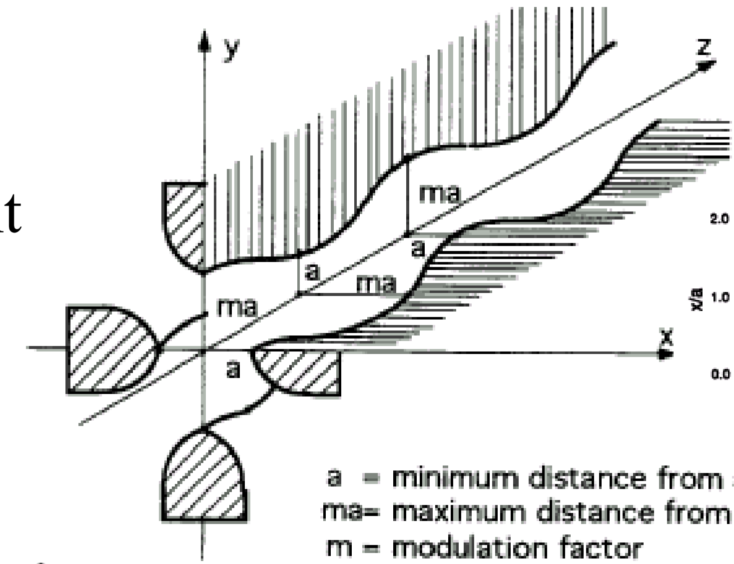


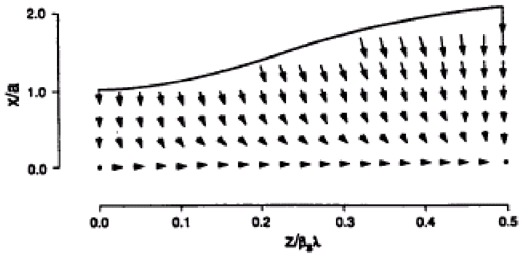
loaded cavity;
mode TE_{210}

It shows an alternating gradient focusing electric field in transverse direction.

Electric focusing –velocity independent

Pole-tip modulation creates a longitudinal electric field component for acceleration.





Near axis, only electric field. Laplace Equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial \vartheta^2}+\frac{\partial^2 U}{\partial z^2}=0$$

The general solution:

$$U(r,\vartheta,z)=\frac{V}{2}[\sum^{n} A_{on}\, r^{2n}\cos 2n\vartheta+\sum^{n}\sum^{l} A_{ln}\, I_{2n}(lkr)\cos 2n\vartheta\, \cos lkz]$$

The lowest order considered:

$$U(r,\vartheta,z)=\frac{V}{2}\left[A_{01}\, r^2\cos 2\vartheta + A_{10} I_0(kr)\cos kz\right]$$

with $$A_{10}=\frac{m^2-1}{m^2 I_0(ka)+I_0(mka)}\ ,$$

$$A_{01}=\frac{1}{a^2}\left[1-A_{10}I_0(ka)\right]=\frac{\chi}{a^2}$$

$$\chi V + A_{10} I_0(ka) V = V$$

χ-focusing efficiency A_{10}- accelerating efficiency

$$E_r = -\frac{\partial U}{\partial r} = -\frac{V}{2}\left[2A_{01}\, r \cos 2\vartheta + kA_{10} I_1(kr) \cos kz\right]$$

$$E_\vartheta = -\frac{1}{r}\frac{\partial U}{\partial \vartheta} = V A_{01} r \sin 2\vartheta$$

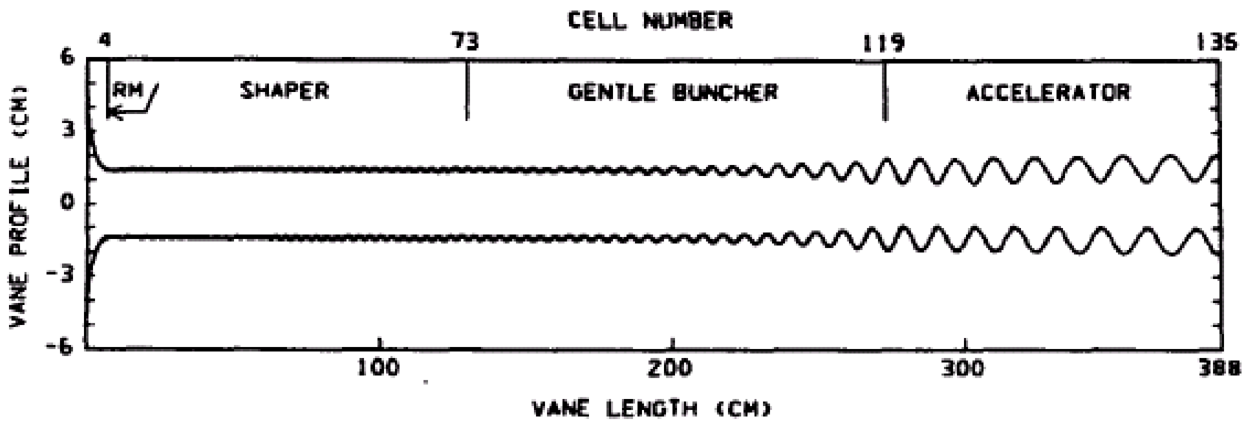
$$E_z = -\frac{\partial U}{\partial z} = \frac{V}{2} kA_{10} I_0(kr) \sin kz \ .$$

$\Delta Ws = qE_0 T l \cos \phi_s$,where $E_0 = 2A_{10}V/ \beta\lambda$

$\beta\uparrow \rightarrow E_0\downarrow$, not good for high energy

4-section design:

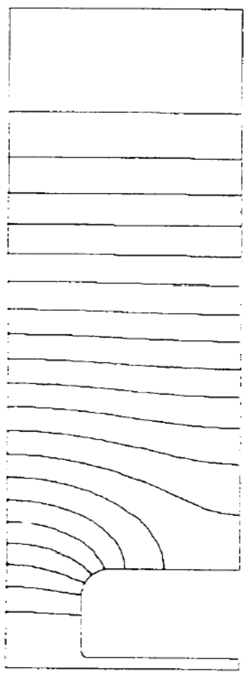
- Radial Matching
- Shaper
- Gentle Bunching
- Acceleration

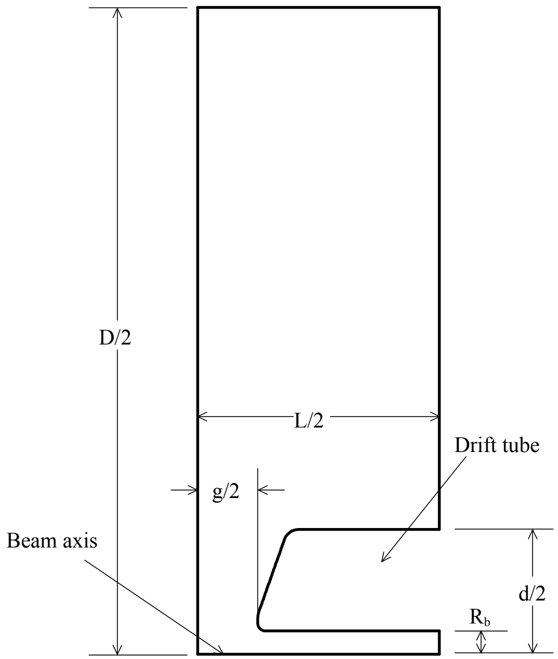


4, Design codes for linac structures

2D code- SUPERFISH in x-y or r-z coordinate systems. It includes several programs for automatically tuning accelerating cavities.

- DTLFISH - DTL tuning code

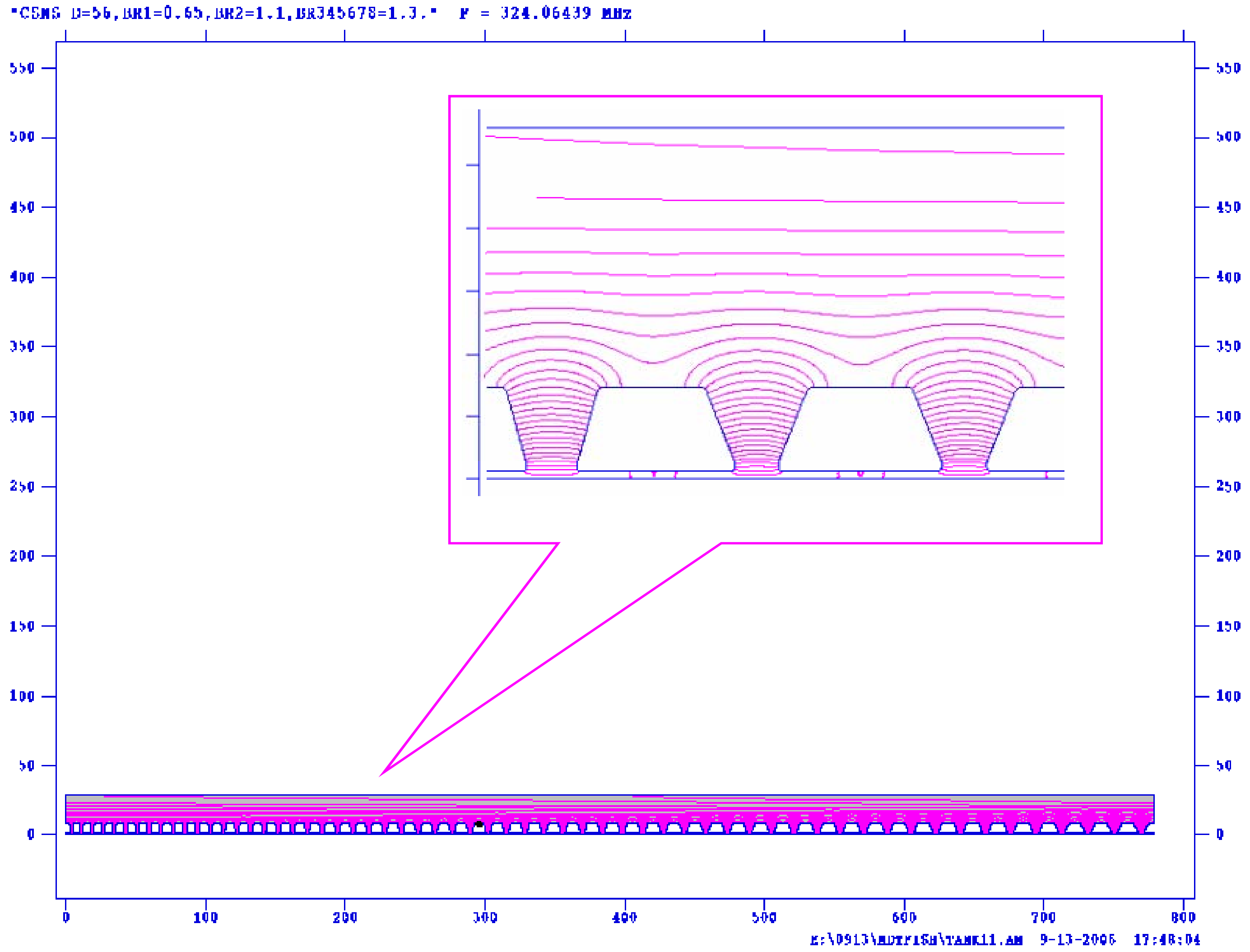




TITLE	
Sample problems for tuning drift-tube linac cells.	
Resonant frequency = 425 MHz	
ENDTITLE	
PLOTTING	Off
PARTICLE	H+
FILEname_prefix	TEST
SEQuence_number	1
FREQuency	425
LENGTH	6.075783
DIAMeter	43
GAP_Length	1.1
E0_Normalization	4.4
CORNER_radius	0.5
INNER_nose_radius	0.325
OUTER_nose_radius	0.325
FLAT_length	0
FACE_angle	5
DRIFT_TUBE_Diameter	8.0594
BEAD_radius	0
GAP_Change	0
STEM_Diameter	1.905
STEM_Count	1
BORE_radius	0.4
PHASE_length	180
DELTA_frequency	0.01
MESH_size	0.05
INCrement	2
START	2

4, Design codes for linac structures

MDTFISH-multi cell DTL modeling



Examples of SUPERFISH cavity tuning code

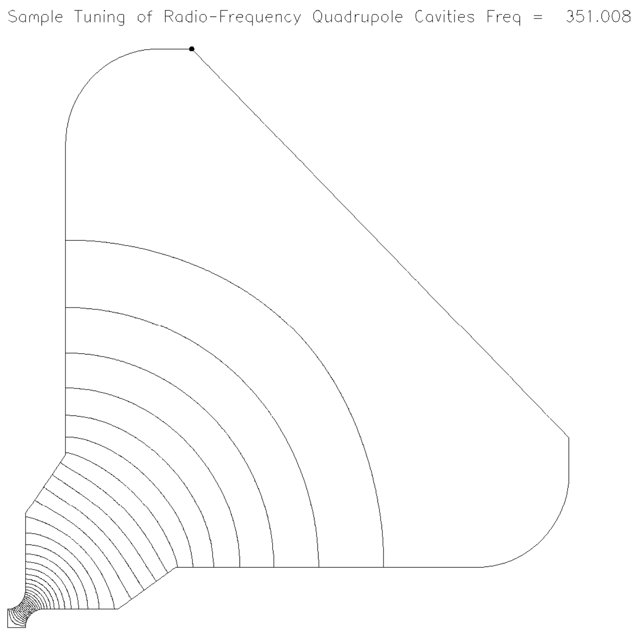
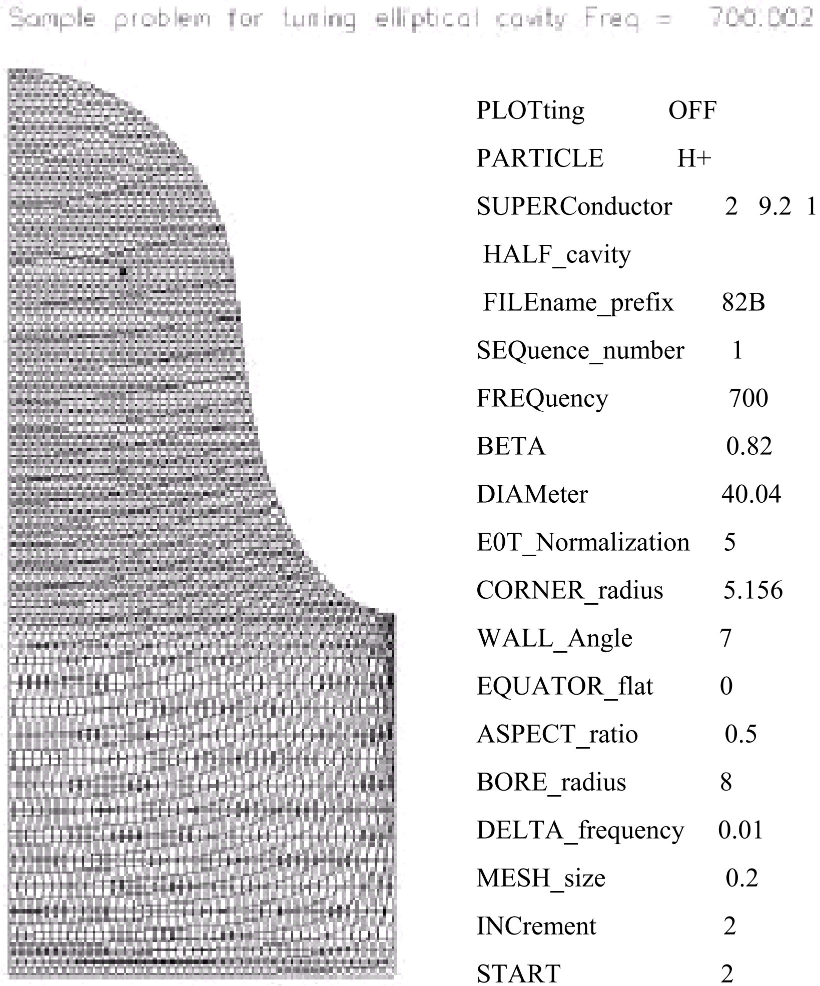


Fig. 18 One quadrant of a RFQ cavity



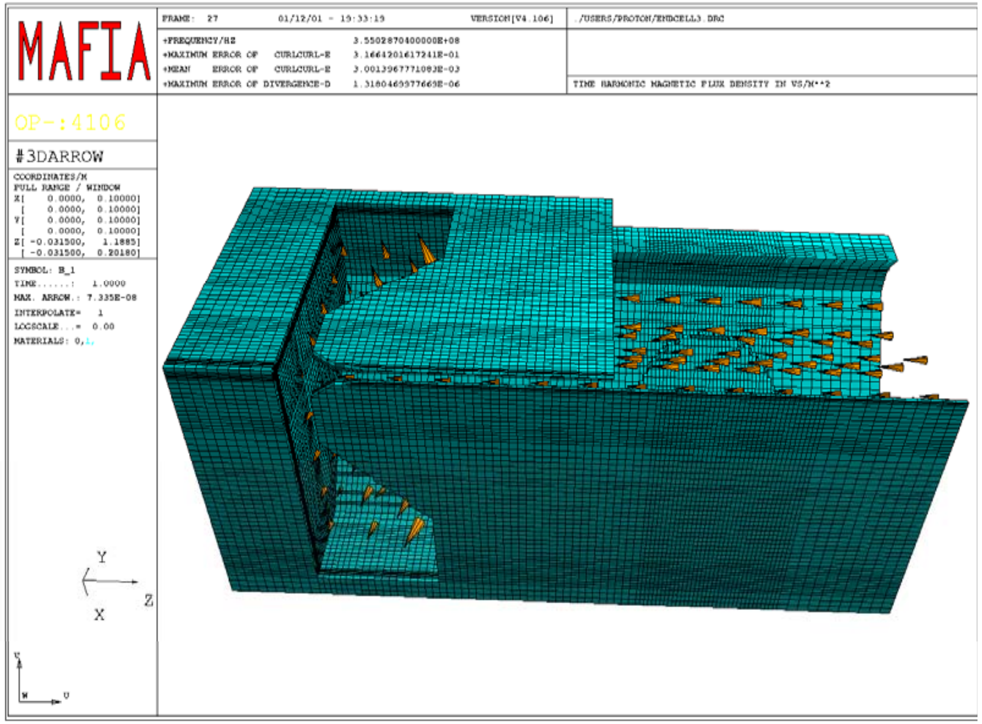
PLOTting	OFF
PARTICLE	H+
SUPERConductor	2 9.2 1.00000E-08
HALF_cavity	
FILEname_prefix	82B
SEQuence_number	1
FREQuency	700
BETA	0.82
DIAMeter	40.04
E0T_Normalization	5
CORNER_radius	5.156
WALL_Angle	7
EQUATOR_flat	0
ASPECT_ratio	0.5
BORE_radius	8
DELTA_frequency	0.01
MESH_size	0.2
INCrement	2
START	2

FTP: PC-AOT-1.ATDIV.LANL.GOV

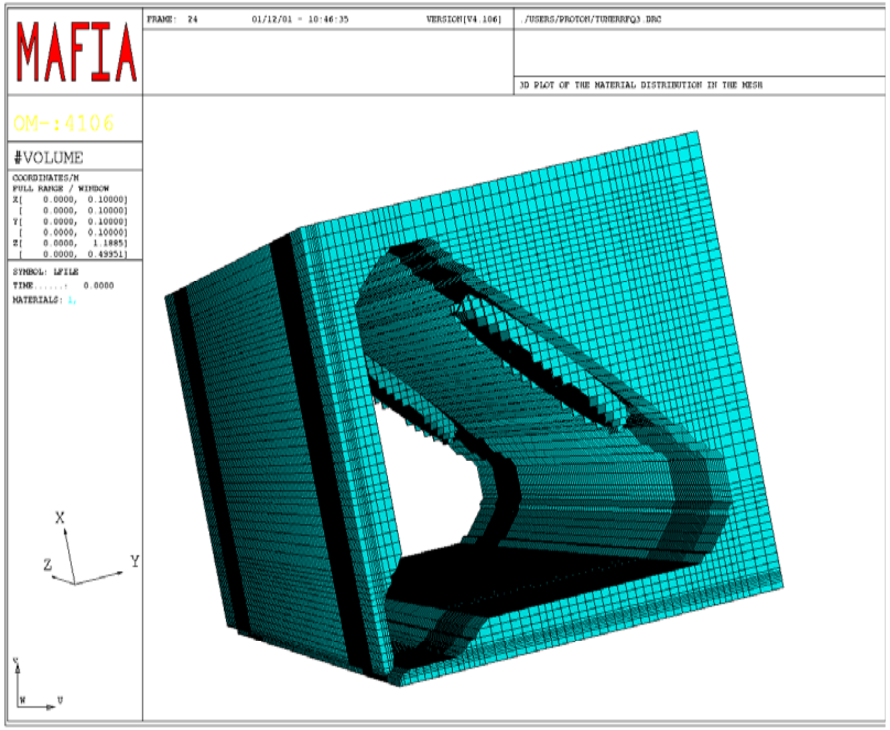
User name: SFUSER

Password: ftpsuperfish

3D code-MAFIA in x-y-z or r-ϕ-z coordinate system

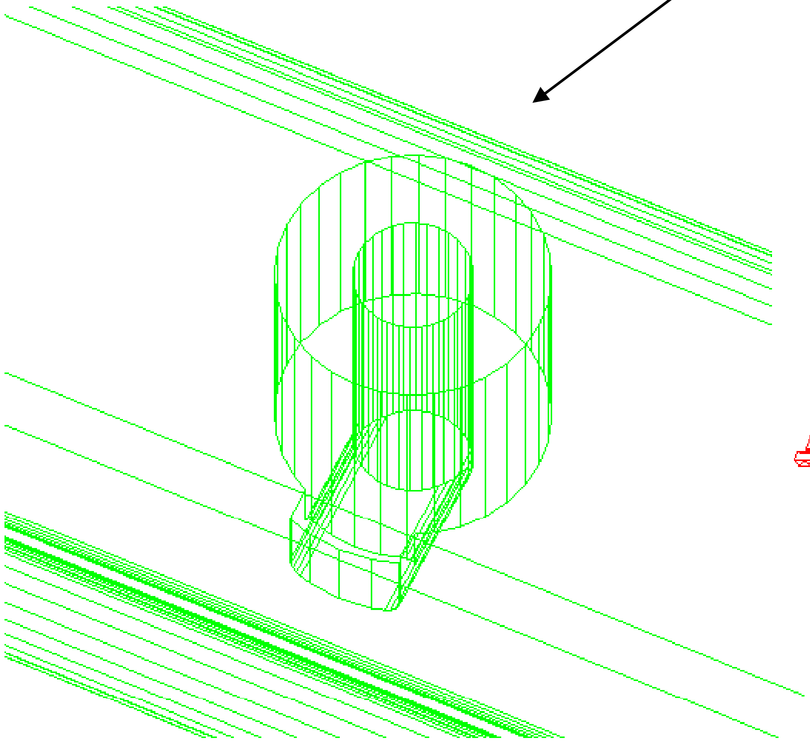


RFQ end region

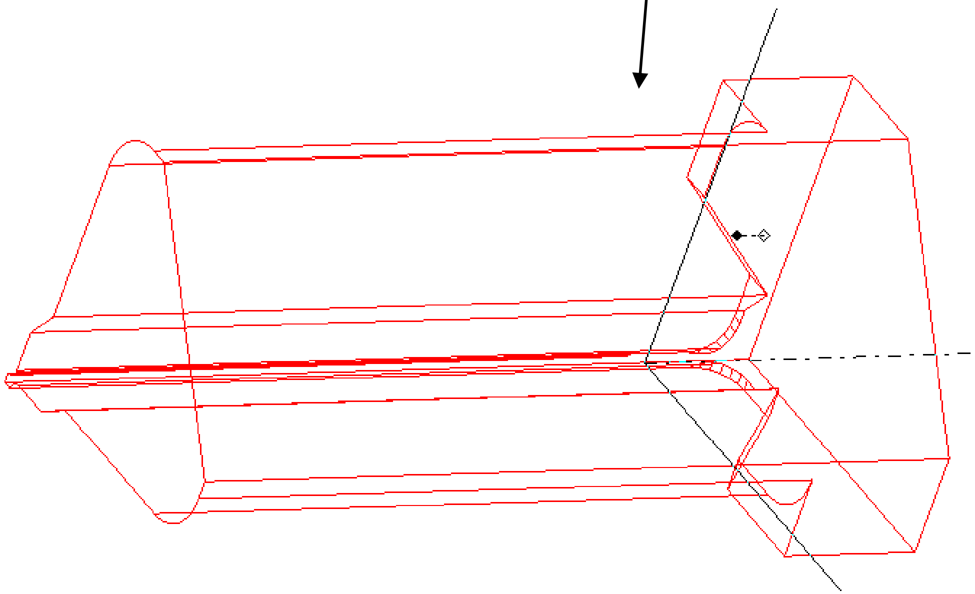


RFQ cavity with tuner

3D code-HFSS for microwave transmission and cavity resonant mode



RFQ input coupler.



RFQ end region

The code gives S parameters
and its spectrum, impedance
or in form of the Smith Chart.

Web: www.ansoft.com

SUMMERY

1, Floquet Theorem: one period difference of field is exp(jk_0d) in a periodic structure.

$$\beta_n = \frac{\omega}{k_n c} = \frac{\beta_0}{1+(n\beta_0\lambda/d)}$$

2, Structure mode: $k_0 d=0,\pi/2,\ldots$

3, Effective shunt impedance: $ZT^2 = \frac{(E_0T)^2}{P/L}$.

4, Alvarez DTL

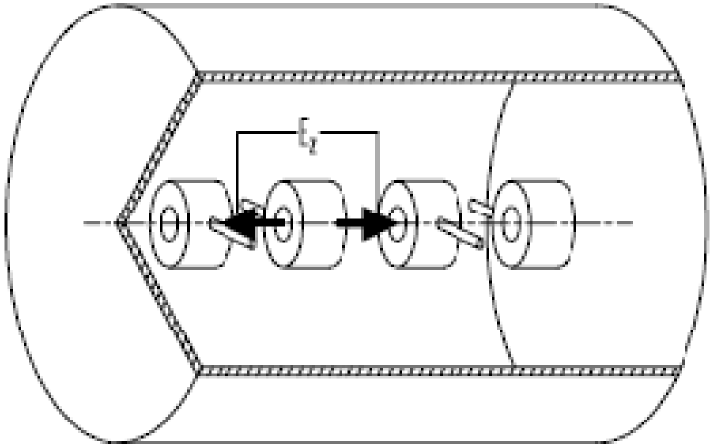
5, Coupled-cavity linacs

6, Superconducting linac

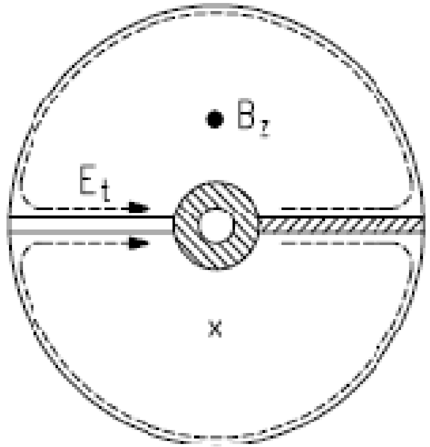
7, RFQ linac

8, Codes: SUPERFISH, MAFIA, HFSS

IH type structure (Interdigital H-mode)



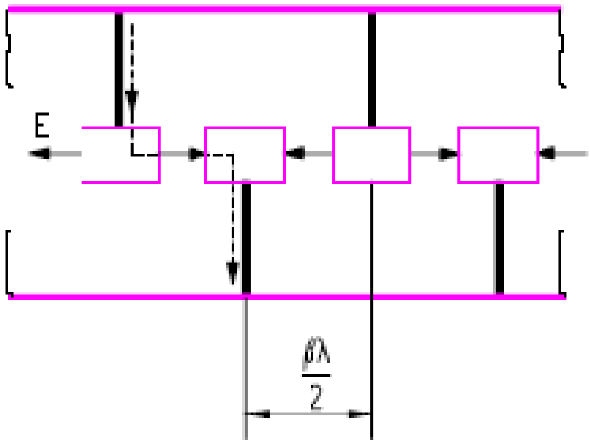
a.)

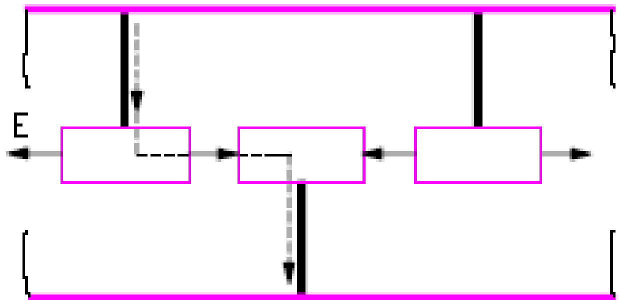


b.)

TE_{110}

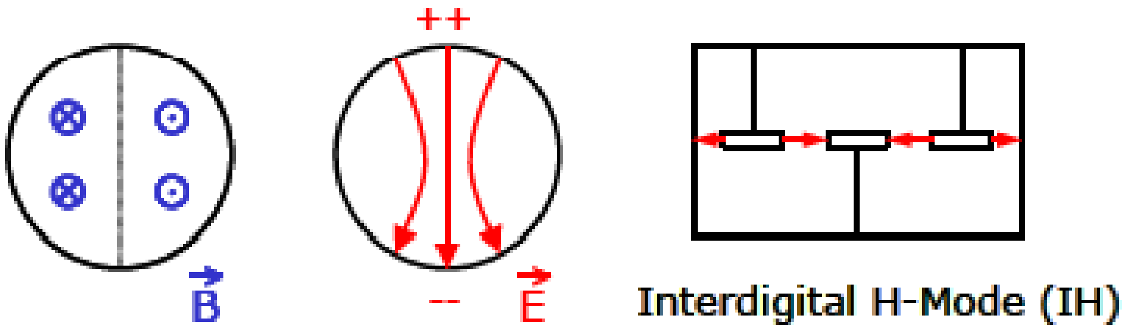
π-mode





The IH structure is very efficient at low beam energies ($\beta \simeq 0.02$ to 0.08) and low RF frequencies (40 to 100 MHz), and is used primarily for ions with $A/Z > 4$. The focusing elements are placed outside between the accelerator tanks.

How to Accelerate Particle with H Type Field?



Empty cylindrical cavity:

1. magnetic field along axis;
2. the time variation of magnetic flux produce electric field transversely ;
3. Electric field located in the whole transverse plan;

Cavity loaded with stem and drift tube:

1. By adding stem and drift tube, the electric field was confined in a limited space (between drift tube) (larger shunt impedance, lower frequency);
2. The transverse electric field was turned along axis by displacing the stem 0.5$\beta\lambda$ along axis ;